IJSRIS journal
INTERNATIONAL JOURNAL OF SCIENTIFIC RESEARCH AND INNOVATIVE STUDIES
ISSN: 2820-7157
www.ijsrisjournal.com

December 2022 Volume 1 Number 3 Pages 29-34

Dislocated Metric Space and Fixed Point Theory for Pairs of Weakly Compatible Mappings

S.K.Jain*, Raj Singh Chandelkar And Shoyeb Ali Sayyed*****

* Professor, Department of Applied Mathematics, Ujjain Engineering College Ujjain (M.P.)India
skjain63engg@gmail.com
** Assistant Professor, Department of Mathematics , Govt. Model College , Jhabua (M.P.) 457661 India
rajchandelkar2910@gmail.com
*** Principal, Royal College of Technology, Indore (M.P.) India
shoyeb9291@gmail.com

ABSTRACT

In present review article, we have generally exposed a unique common fixed point theorem in dislocated metric space with pairs of weakly compatible mappings . For getting our main result , we applied contractive type condition with some beard out lemmas, definitions, and references in concern areas.

Keywords and phrases : Common fixed point theorem, Continuous mappings, Dislocated metric space,Weakly Compatible mapping,.

1. INTRODUCTION AND PRELIMINARIES

Hitzler and Seda [3] developed the idea of dislocated topology in year 2000 with initiation of dislocated metric space came in knowledge . Using this concept many researchers , scientists and mathematicians established , improved and advanced it with number of publications . Ahmed [1], Sayyed et. al [15,16,18,19,20,21] , Sayyed and Badshah [17] , Lateef et. al. [9] and Sarma and Kumari [14] gave some significant result for compatible mappings with certain , contractive and more general condition used and gave contribution for it.

In the similar path Daheriya et. al [2] , Jha and Pathak [8], Mishra and Wadkar [10], Patni [11], Tas et. al [22], Patni and Kumari [12,13] gave more contribution and further enhancement for development of this concept. Recently Jain [4], jain and Sayyed [5,6,7] and Wadkar et al. [23,24] generalized more results for weakly compatibility and fixed point theory.

Definition 1.1: [Zeyada et.al [25]]. Let $d': B \times B \rightarrow [0,\infty)$ be a function where B is a non empty set and following conditions hold :

1.1.1. $d'(p, q) = d'(q, p)$,

1.1.2. $d'(p, q)= d'(q, p) = 0$ implies $p = q$

1.1.3. $d'(p, q) \leq d'(p, r) + d'(r, q)$ for all $p, q, r \in B$.

Then d' is called dislocated metric (or simply d'-metric) on B.

2. MAIN RESULT

Theorem 2.1 : Let E,F,M and N be continuous mappings from B into itself, where B is complete d'-metric space along with

(C-1) N(B) C E(B) and M(B) C F(B),

(C-2) The pairs (M,E) and (N,F) are weakly compatible and,

(C-3)

$$d'(Mp,Nq) \leq a \frac{d'(Ep,Mp)d'(Fq,Nq)+d'(Ep,Nq)d'(Fq,Mp)}{d'(Ep,Fq)}$$

$$+ b \frac{d'(Ep,Nq)[d'(Ep,Mp+d'(Fq,Nq))]}{d'(Ep,Fq)+d'(Fq,Nq)+d'(Fq,Mp)}$$

$$+ c \frac{d'(Ep,Mp)d'(Fq,Mp)+ d'(Fq,Nq)d'(Ep,Nq)}{d'(Fq,Mp)+d'(Ep,Nq)}$$

$$+ d [d'(Ep,Mp) + d'(Fq,Nq)]$$

$$+ e [d'(Fq,Mp)+d'(Ep,Nq)] + f d'(Ep,Fq)$$

For all $p,q \in B$ and a , b ,c ,d, e and f are positive with $0 \leq a+2b+c+2d+2e+f \leq 1$. Then E, F, M and N have a unique common fixed point.

PROOF . Using condition (C-1), we define the sequences $\{p_n\}$ and $\{q_n\}$ in B such that

$$q_{2n} = Fp_{2n+1} = Mp_{2n}$$

and $\quad q_{2n+1} = Ep_{2n+2} = Np_{2n+1}$, $\quad n = 0,1,2,---$

CASE I - If $q_{2n} = q_{2n+1}$ for some n then $Fp_{2n+2} = Np_{2n+1}$ therefore p_{2n+1} is coincidence point of F and N also if $q_{2n+1} = q_{2n+2}$ for some n then $Ep_{2n+2} = Mp_{2n+2}$. Hence p_{2n+2} is a coincidence point of E and M.

CASE II - Now we assume $q_{2n} \neq q_{2n+1}$, $\forall$ n ,

we have

$d'(q_{2n} , q_{2n+1}) = d'(Mp_{2n},Np_{2n+1})$, then using condition (C-3) ,

$$\leq a \frac{d'(Ep_{2n},Mp_{2n})d'(Fp_{2n+1},Np_{2n+1})+d'(Ep_{2n},Np_{2n+1})d'(Fp_{2n+1},Mp_{2n})}{d'(Ep_{2n},Fp_{2n+1})}$$

$$+ b \frac{d'(Ep_{2n},Np_{2n+1})[d'(Ep_{2n},Mp_{2n})+d'(Fp_{2n+1},Np_{2n+1})]}{d'(Ep_{2n},Fp_{2n+1})+d'(Fp_{2n+1},Np_{2n+1})+d'(Fp_{2n+1},Mp_{2n})}$$

$$+ c \frac{d'(Ep_{2n},Mp_{2n})d'(Fp_{2n+1},Mp_{2n})+ d'(Fp_{2n+1},Np_{2n+1})d'(Ep_{2n},Np_{2n+1})}{d'(Fp_{2n+1},Mp_{2n})+d'(Ep_{2n},Np_{2n+1})}$$

$$+ d [d'(Ep_{2n},Mp_{2n}) + d'(Fp_{2n+1},Np_{2n+1})]$$

$$+ e [d'(Fp_{2n+1},Mp_{2n})+d'(Ep_{2n},Np_{2n+1})] + fd'(Ep_{2n},Fp_{2n+1})$$

$$d'(q_{2n} , q_{2n+1}) \leq a \frac{d'(q_{2n-1}, q_{2n})d'(q_{2n},q_{2n+1})+ d'(q_{2n-1},q_{2n+1})d'(q_{2n},q_{2n})}{d'(q_{2n-1},q_{2n})}$$

$$+ b \frac{d'(q_{2n-1},q_{2n+1})[d'(q_{2n-1},q_{2n})+ d'(q_{2n},q_{2n+1})]}{d'(q_{2n-1},q_{2n})+ d'(q_{2n},q_{2n+1})+ d'(q_{2n},q_{2n})}$$

$$+ c \frac{d'(q_{2n-1},q_{2n})d'(q_{2n},q_{2n})+ d'(q_{2n},q_{2n+1}) d'(q_{2n-1},q_{2n+1})}{d'(q_{2n},q_{2n})+d'(q_{2n-1},q_{2n+1})}$$

$$+ d[d'(q_{2n-1},q_{2n}) + d'(q_{2n}, q_{2n+1})] + e [d'(q_{2n}, q_{2n})$$

$$+ d'(q_{2n-1}, q_{2n+1})] + f d'(q_{2n-1}, q_{2n})$$

$$d'(q_{2n}, q_{2n+1}) \leq a d'(q_{2n}, q_{2n+1})] + b\, d(q_{2n-1}, q_{2n+1})$$

$$+ c\, d'(q_{2n}, q_{2n+1}) + d [d'(q_{2n-1},q_{2n}) + d'(q_{2n}, q_{2n+1})]$$

$$+ e\, d'(q_{2n-1}, q_{2n+1}) + f d'(q_{2n-1}, q_{2n})$$

Or

$$d'(q_{2n}, q_{2n+1}) \leq \frac{b+d+e+f}{1-a-b-c-d-e} d'(q_{2n-1}, q_{2n})$$

Or

$d'(q_{2n}, q_{2n+1}) \leq u\ d'(q_{2n-1}, q_{2n})$,

where $u = \frac{b+d+e+f}{1-a-b-c-d-e}$.

This shows that

$d'(q_n, q_{n+1}) \leq u\, d'(q_{n-1}, q_n) \leq ... \leq u^n\ d'(q_0, q_1)$. for any integer $n > 0$, we have

$$d'(q_n, q_{n+q}) \leq d'(q_n, q_{n+1}) + d'(q_{n+1}, q_{n+2}) + d'(q_{n+2}, q_{n+3}) + --- + d'(q_{n+q-1}, q_{n+q})$$
$$\leq (1+u+u^2+---+u^n)\, d'(q_n, q_{n+1})$$

$$d'(q_n, q_{n+q}) \leq \frac{u^n}{1-u} d'(q_0, q_1) .$$

Since $0 < u < 1$, $u^n \to 0$ as $n \to \infty$, so we get

$d'(q_n, q_{n+q}) \to 0$.

By Hitzler and Seda [3] it's clear that $\{q_n\}$is a Cauchy sequence. Now ∃ a point r in B implies $\{q_n\} \to r$

subsequences $\{Mp_{2n}\} \to r$, $\{Fp_{2n+1}\} \to r$, $\{Np_{2n+1}\} \to r$, $\{Ep_{2n+2}\} \to r$.

Since N(B) C E(B) , then ∃ a point u* in B such that $r = Eu^*$, so

$$d'(Mu^*,r) \leq d'(Mu^*,Np_{2n+1})$$
$$\leq a \frac{d'(Eu^*,Mu^*)d'(Fp_{2n+1},Np_{2n+1})+d'(Eu^*,Np_{2n+1})d'(Fp_{2n+1},Mu^*)}{d'(Eu^*,Fp_{2n+1})}$$
$$+ b \frac{d'(Eu^*,Np_{2n+1})[d'(Eu^*,Mu^*)+d'(Fp_{2n+1},Np_{2n+1})]}{d'(Eu^*,Fp_{2n+1})+d'(Fp_{2n+1},Np_{2n+1})+d'(Fp_{2n+1},Mu^*)}$$
$$+ c \frac{d'(Eu^*,Mu^*)d'(Fp_{2n+1},Mu^*)+ d'(Fp_{2n+1},Np_{2n+1})d'(Eu^*,Np_{2n+1})}{d'(Fp_{2n+1},Mu^*)+d'(Eu^*,Np_{2n+1})}$$
$$+ d\ [d'(Eu^*,Mu^*) + d'(Fp_{2n+1},Np_{2n+1})]$$
$$+ e\ [d'(Fp_{2n+1},Mu^*)+d'(Eu^*,Np_{2n+1})] + fd'(Eu^*,Fp_{2n+1})$$

Taking limit as $n \to \infty$ and using condition , we get

$$d'(Mu^*,r) \leq (c+d+e)\, d'(Mu^*,r)) ,$$

a contradiction hence clearly $Mu^* = Eu^* = r$.

Now M(B) C F(B) , then ∃ a point v* in B such that $r = Fv^*$, we claim that $r = Nv^*$.

If $r \neq Nv^*$, then

$$d'(r,Nv^*) = d'(Mu^*,Nv^*)$$
$$\leq a \frac{d'(Eu^*,Mu^*)d'(Fv^*,Nv^*)+d'(Eu^*,Nv^*)d'(Fv^*,Mu^*)}{d'(Eu^*,Fv^*)}$$
$$+ b \frac{d'(Eu^*,Nv^*)[d'(Eu^*,Mu^*)+d'(Fv^*,Nv^*)]}{d'(Eu^*,Fv^*)+d'(Fv^*,Nv^*)+d'(Fv^*,Mu^*)}$$
$$+ c \frac{d'(Eu^*,Mu^*)d'(Fv^*,Mu^*)+ d'(Fv^*,Nv^*)d'(Eu^*,Nv^*)}{d'(Fv^*,Mu^*)+d'(Eu^*,Nv^*)}$$
$$+ d\ [d'(Eu^*,Mu^*) + d'(Fv^*,Nv^*)]$$
$$+ e\ [d'(Fv^*,Mu^*)+d'(Eu^*,Nv^*)] + f\ d'(Eu^*,Fv^*)$$

$$d'(r,Nv^*) \leq (b+c+d+e)\, d'(r,Nv^*)$$

clearly $r = Nv^*$. Hence we claim that

$$Mu^* = Eu^* = Nv^* = Fv^* = r .$$

Since the pair (M,E) is weakly compatible so by definition

$$MEu^* = EMu^* \Rightarrow Mr = Er.$$

we will demonstrate r is a fixed point of M. If we take $Mr \neq r$, then

$$d'(r,Mr) = d'(Mr,Nv^*)$$
$$\leq a \frac{d'(Er,Mr)d'(Fv^*,Nv^*)+d'(Er,Nv^*)d'(Fv^*,Mr)}{d'(Er,Fv^*)}$$
$$+ b \frac{d'(Er,Nv^*)[d'(Er,Mr)+d'(Fv^*,Nv^*)]}{d'(Er,Fv^*)+d'(Fv^*,Nv^*)+d'(Fv^*,Mr)}$$
$$+ c \frac{d'(Er,Mr)d'(Fv^*,Mr)+ d'(Fv^*,Nv^*)d'(Er,Nv^*)}{d'(Fv^*,Mr)+d'(Er,Nv^*)}$$
$$+ d\ [d'(Er,Mr) + d'(Fv^*,Nv^*)]$$
$$+ e\ [d'(Fv^*,Mr)+d'(Er,Nv^*)] + f\ d'(Er,Fv^*)$$

Or

$$d'(r,Mr) \leq (a+2e+f)\, d'(r,Mr)$$

this is a contradiction hence $r = Mr$. Hence we have $Er = Fr = Mr = Nr = r$, this shows that r is a common fixed point of E, F, M and N.

For uniqueness : Let $u^* \neq v^*$ be two common fixed points of the mappings E, F, M and N, then we have

$$d'(u^*,v^*) = d'(Mu^*,Nv^*)$$

$$\leq a \frac{d'(Eu^*,Mu^*)d'(Fv^*,Nv^*)+d'(Eu^*,Nv^*)d'(Fv^*,Mu^*)}{d'(Eu^*,Fv^*)}$$

$$+ b \frac{d'(Eu^*,Nv^*)[d'(Eu^*,Mu^*)+d'(Fv^*,Nv^*)]}{d'(Eu^*,Fv^*)+d'(Fv^*,Nv^*)+d'(Fv^*,Mu^*)}$$

$$+ c \frac{d'(Eu^*,Mu^*)d'(Fv^*,Mu^*)+ d'(Fv^*,Nv^*)d'(Eu^*,Nv^*)}{d'(Fv^*,Mu^*)+d'(Eu^*,Nv^*)}$$

$$+ d [d'(Eu^*,Mu^*) + d'(Fv^*,Nv^*)]$$

$$+ e [d'(Fv^*,Mu^*)+d'(Eu^*,Nv^*)] + f d'(Eu^*,Fv^*)$$

$d'(u^*,v^*) \leq (a+2e+f) d'(u^*,v^*)$ a contradiction this shows that $d'(u^*,v^*) = 0$, since d' is a dislocated metric space then by definition $u^* = v^*$.

this complete the proof .

COROLLARY 2.1: Let M and N be continuous mappings of B into itself and (B, d') be a complete d'-metric, satisfying

$$d'(Mp,Nq) \leq a \frac{d'(p,Mp)d'(q,Nq)+d'(p,Nq)d'(q,Mp)}{d'(p,q)}$$

$$+ b \frac{d'(p,Nq)[d'(p,Mp+d'(q,Nq))]}{d'(p,q)+d'(q,Nq)+d'(q,Mp)}$$

$$+ c \frac{d'(p,Mp)d'(q,Mp)+ d'(q,Nq)d'(p,Nq)}{d'(Fq,Mp)+d'(Ep,Nq)}$$

$$+ d [d'(p,Mp) + d'(q,Nq)] + e [d'(q,Mp)+d'(p,Nq)] + f d'(p,q)$$

For all p,q ϵ B and a , b ,c ,d and e are greater than zero with $0 \leq a+2b+c+2d+2e+f \leq 1$ with M and N have a unique common fixed point.

PROOF : Taking E = F = *I* (identity mapping)in previous proved terms and process then we can establish this corollary

COROLLARY 2.2: Let N be a continuous mapping of B into itself and (B, d') be a complete d'-metric satisfying

$$d'(Np,Nq) \leq a \frac{d'(p,Np)d'(q,Nq)+d'(p,Nq)d'(q,Np)}{d'(p,q)}$$

$$+ b \frac{d'(p,Nq)[d'(p,Np+d'(q,Nq))]}{d'(p,q)+d'(q,Nq)+d'(q,Np)} + c \frac{d'(p,Np)d'(q,Np)+ d'(q,Nq)d'(p,Nq)}{d'(Fq,Np)+d'(Ep,Nq)}$$

$$+ d [d'(p,Np) + d'(q,Nq)] + e [d'(q,Np)+d'(p,Nq)] + f d'(p,q)$$

For all p,q ϵ B and a , b ,c ,d and e are greater than zero with $0 \leq a+2b+c+2d+2e+f \leq 1$ with N has a unique fixed point.

PROOF : Assuming E = F = M = *I* (identity mapping) in previous part and process then we can establish this corollary .

COROLLARY 2.3 : Let M and N be continuous mappings of B into itself and (B, d') be a complete d'-metric, satisfying

$$d'(Mp,Nq) \leq a \frac{d'(p,Mp)d'(q,Nq)+d'(p,Nq)d'(q,Mp)}{d'(p,q)} + b \frac{d'(p,Nq)[d'(p,Mp+d'(q,Nq))]}{d'(p,q)+d'(q,Nq)+d'(q,Mp)}$$

$$+ c \frac{d'(p,Mp)d'(q,Mp)+ d'(q,Nq)d'(p,Nq)}{d'(Fq,Mp)+d'(Ep,Nq)} + d [d'(p,Mp) + d'(q,Nq)] + e [d'(q,Mp)+d'(p,Nq)] + f d'(p,q)$$

For all p,q ϵ B and a , b ,c ,d and e are greater than zero with $0 \leq a+2b+c+2d+2e+f \leq 1$ with M and N have a unique common fixed point.

PROOF : If E = N and F = I in previous proved terms and process then we can establish this corollary

CONCLUSION

In our existing main part we have elaborated four mappings with rational type inequality for reclaiming the earlier justified outcome . we have felt that it may be keeping up with other type like changes of spaces, mappings with new ideas.

REFERENCES

[1] M. A. , Ahmed, Common fixed point theorems for weakly compatible mappings, Rocky Mountain J. Math. 33 (4) (2003), 1189–1203.

[2] R. D., Daheriya, R ., Jain, and M., Ughade, Some fixed point theorem for expansive type mapping in dislocated metric space. ISRN Math. Anal. 2012, Article ID 376832 (2012).

[3] P. Hitzler, and, A.K.., Seda Dislocated Topologies. Journal of Electrical Engineering,51, (2000) 3-7.

[4] S.K.., Jain .,Outcome of a unique fixed point theorem by using rational inequality, Journal of Emerging Technologies and Innovative Research , Volume 6, Issue 5, (2019)139-143

[5] S.K. Jain , and S. A., Sayyed , Weak compatibility for four mappings and general common fixed point theorem " , International Journal of Research and Analytical Reviews , Vol**.** 6, Issue 1, (2019),990-994.

[6] S.K. Jain , and S. A., Sayyed , Nonlinear contraction satisfying square inequality and fixed point theory", International Journal of Research and Analytical Reviews , Vol. 6, Issue 2, (2019),923-929.

[7] S.K. Jain , and S. A., Sayyed , Some Prosperous Outcomes In 2- Banach Space For Fixed And Common Fixed Point Theorems, Poonam Shodh Rachna, Vol. 1, Issue.V, (2022),19-26.

[8] K., K., Jha, and D., Panths, common fixed point theorem in dislocated metric spaces, Applied mathematical sciences vol .6 , no. 91,(2012) 4497-4503.

[9] D., Lateef, S.A,. Sayyed, And A., Bhattacharyya, "Common Fixed Point For Multivalued And Comatible Maps." Ultra Scientist Vol. 21 (2)M, (2009).503-508

[10] L.N ., Mishra,.and B.R., Wadkar, Dislocated Metric Space with Some Fixed Point Theorems, International Journal of Scientific and Innovative Mathematical Research , *Vol.7, Issue3, ,(2019).9-24*.

[11] D. Panthi, ,Fixed Point Results in Cyclic Contractions of Generalized Dislocated Metric Spaces. Annals of Pure and Applied Mathematics , 5, (2014) 192-197.

[12] D Panthi, and S., Kumari,., Some Common Fixed Point Theorems for Four Mappings in Dislocated Metric Space, Advances in Pure Mathematics Vol.6 No.10, (2016), 695-712.

[13] D. Panthi, and P.S. Kumari, Some Integral Type Fixed Point Theorems in Dislocated Metric Space. American Journal of Computational Mathematics , 6, (2016) 88-97.

[14] I.R., Sarma, and P.S., Kumari, On dislocated metric spaces. Int. J. Math. Arch. 3(1), (2012). 72-77

[15] S.A., Sayyed "Some Results On Common Fixed Point For Multivalued And Compatible Maps",Ultra Engineer , Vol.1 (2), (2012).191-194.

[16] S.A. , Sayyed , Fuzzy metric space and sequel of common fixed point Theorem using property E.A., JMSCM, Vol.3, No.2, (2022), 218-224.

[17] S.A. Sayyed, And . V.H., Badshah,. "Common Fixed Points For Compatible And Multivalued Mappings", Varahmihir Journal Of Mathematical Sciences Vol. 6, No 2(2006).,613-618 .

[18] S.A., Sayyed, K.., Devghare, and V.H., Badshah, A Note On Fixed Point For Selfmaps", Acta Ciencia Indica", Vol. XXXII M, No. 4, 1595-1596, (2006).

[19**]** S.A., Sayyed, F., Sayyed,. and V.H. Badshah, ,Fixed Points Theorem And Multivalued Mappings, *Acta Ciencia Indica, Vol. XXVIII M,No. 2, (2002) 155-158*

[20] S.A. Sayyed, and F., Sayyed, Some results on fixed point theorems by altering distances between the points " Oriental journal of physical sciences Vol.1 (No.1&2), (2016) 23-26.

[21] Sayyed,S.A., Vyas,L., Sayyed,F. And Badshah,V.H., "Some Results On Fixed Point Theorems For Selfmaps" Ultra Engineer , Vol.1 (2), (2012).143-145.

[22] K., Tas., M. Telci, and B., Fisher, Common fixed point theorems for compatible mapping, Int. J. math. Math. Sci., 19(3)(1996), 451-456.

[23] B.R., Wadkar, R .,Bhardwaj,.and ,B., Singh A common fixed point theorem in dislocated metric space, International Journal of Engineering Research and Development, *Volume 10, Issue 6 (2014),12*-17

[24] B.R., Wadkar, R Bhardwaj,.and B., Singh, Some Fixed Point Theorems in Dislocated Metric Space, Global Journal of Pure and Applied Mathematics. Volume 13, Number 6 (2017), 2089-2110.

[25] F.M., Zeyada, G.H., Hassan, and M.A. Ahmed, "A Generalization of a fixed point theorem due toHitzler and Seda in dislocated quasi-metric spaces", The Arabian J. Sci. Engg., **31** (1A)(2006), 111-11