

## **Weakly Compatible Mappings Along With Property E.A. In Fuzzy Metric Space And Common Fixed Point Theorem**

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### **ABSTRACT**

The present review article is an enhancement of a common fixed point theorem in fuzzy metric space for a pair of weakly compatible mappings along with property E.A. . In this article we have extended, established and improved result with the help of a contractive type condition and altering distance function .

**KEYWORDS AND PHRASES:** Altering Distance, E.A. Property, Fixed Point Theorem, Fuzzy Metric Space , Weakly Compatible

### **1. INTRODUCTION**

Fuzzy Mathematics is a very interesting branch of mathematics ,in which contains fuzzy set theory as well as fuzzy logic conceptualization of fuzzy sets .

It was first set up by mathematician Prof. Lotfi A. Zadeh [34] . Recently Sayyed [21] established a sequel of common fixed point Theorem using property E.A. . Proceeding the above concept with referring it Kramosil and Michalek [13], Pathak et.al [17],Grabiec[6], Imdad, Ali and Hasan [8],Mihet [16], Sayyed [20] ,Jain and Sayyed [9,10], Sastry,Naidu and Krishn [19], Bratney and Odeh [15],Romaguera ,Sapena and Tirado [18],Sayyed [20], Shirude and Aage [29], Steimann [30],Sayyed and Badshah [22], Sayyed and Sayyed [24], Sayyed et al [23,25], Subrahmanyam [31], Jungck [11], Amari and Moutawakil [1], Sedghi,et.al.[27], Khan [12], Fuller [3], Shen,et.al.[28], Wairojjana, et.al. [33] and Manthena and Manchala [14], Vijayaraju and Sajath [32], using various conditions and inequalities for getting the significant result .

## 2. PRELIMINARIES

**Definition 2.1.** [Schweizer and Sklar [26]] :

A binary operation  $*$  from  $[0,1] \times [0,1]$  to  $[0,1]$  is a continuous  $z$ -norm if for all  $a, a', b, b' \in z$  and

$$(2.1.1) \quad a * 1 = a,$$

$$(2.1.2) \quad a * a' = a' * a,$$

$$(2.1.3) \quad a * a' \leq b * b' \text{ whenever}$$

$$a \leq b \text{ and } a' \leq b',$$

$$(2.1.4) \quad a * (a' * b) = (a * a') * b.$$

**Definition 2.2** [George and Veeramani [4]] : If  $W$  is an arbitrary set,  $*$  is a continuous  $z$ -norm and  $M$  is a fuzzy set in  $W \times W \times (0, \infty)$  then the 3-tuple  $(W, M, *)$  is called a fuzzy metric space, if

$$(2.2.1) \quad M(x, y, z) > 0,$$

$$(2.2.2) \quad M(x, y, z) = 1$$

for all  $z > 0$  if and only if  $x = y$ ,

$$(2.2.3) \quad M(x, y, z) = M(y, x, z),$$

$$(2.2.4) \quad M(x, c, z + s) \leq M(x, y, z) * M(y, c, s),$$

$$(2.2.5) \quad M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$$

is a continuous function, for all  $x, y, c \in W$  and  $z, s > 0$ .

For obtaining our result we refer the definition of E. A. property by Abbas et.al [2] and altering distance function by Zadeh [34].

## 3. MAIN RESULTS

**Theorem 3.1.** Let  $(W, M, *)$  be a fuzzy metric space and two weakly compatible self mappings  $H$  and  $G$  of  $W$  into itself with satisfying the condition 3.1.1 which is

$$(3.1.1)$$

$$\begin{aligned} \phi(M(Hx, Hy, z)) \leq & \phi \left\{ \alpha_1(z) \frac{M(Gx, Hx, z)M(Gy, Hy, z)}{M(Gx, Gy, z)} \right. \\ & + \alpha_2(z) \frac{M(Gx, Hy, z)M(Gy, Hx, z)}{M(Gx, Gy, z)} + \\ & \alpha_3(z)M(Hx, Gx, z) + \alpha_4(t) M(Gy, Hy, t) \\ & + \alpha_5(t)M(Gx, Gy, t) + \\ & \left. \alpha_6(z)M(Hx, Gy, 2z) \right\} \end{aligned}$$

Where  $x, y \in W$ ,  $\alpha_i$  from  $(0, \infty)$  to  $(0, 1)$ , where  $i = 1$  to 6 and  $z > 0$  and altering distance function is  $\phi$ . If the property E.A. satisfied by  $H$  and  $G$  with range of  $G$  is a closed subspace of  $W$ , then  $H$  and  $G$  have a unique common fixed point in  $W$ .

**Proof:** Property E.A. satisfied by  $H$  and  $G$  then  $\exists$  a sequence  $\{x_n\}$  in  $W$  as

$$\lim_{n \rightarrow \infty} Hx_n = \lim_{n \rightarrow \infty} Gx_n = c \in X \quad \text{--- 3.1.2}$$

We have  $G(x)$  is a closed subspace of  $W$ ,  $\exists u \in W$  such that  $c = Gu$  --- 3.1.3

If for  $x = x_n, y = u$ , Equation 3.1.1 becomes,

$$\begin{aligned} \phi(M(Hx_n, Hu, z)) & \leq \\ \phi \left\{ \alpha_1(z) \frac{M(Hx_n, Hx_n, z)M(Gu, Hu, z)}{M(Gx_n, Gu, z)} + \right. \\ & \alpha_2(z) \frac{M(Gx_n, Hu, z)M(Gu, Hx_n, z)}{M(Gx_n, Gu, z)} \\ & + \alpha_3(z) M(Hu, Gx_n, z) + \alpha_4(z) M(Gu, Hu, z) + \\ & \left. \alpha_5(z)M(Gx_n, Gu, z) + \alpha_6(z) M(Hx_n, Gu, 2z) \right\} \end{aligned}$$

Taking Limit  $n \rightarrow \infty$  and using Equations 3.1.2, 3.1.3, we get

$$\begin{aligned} \phi(M(c, Hu, z)) & \leq \phi \left\{ \alpha_1(z) \frac{M(c, c, z)M(c, Hu, z)}{M(c, c, z)} \right. \\ & + \alpha_2(z) \frac{M(c, Hu, z)M(c, c, z)}{M(c, c, z)} + \alpha_3(z) M(Hu, c, z) \\ & + \alpha_4(z)M(c, Hu, z) + \alpha_5(z)M(c, c, z) \\ & \left. + \alpha_6(z) M(c, c, 2z) \right\} \\ \phi(M(c, Hu, z)) & \leq \phi \left\{ \alpha_1(z) M(c, Hu, z) \right. \\ & \left. + \alpha_2(z) M(c, Hu, z) + \alpha_3(z) M(Hu, c, z) \right\} \end{aligned}$$

$$+ \alpha_4(z) M(c, Hu, z) \} + \alpha_5(z)\phi(1)+ \alpha_6(z) \phi(1)$$

Or

$$\phi(M(c, Hu, z)) = 0 \text{ which implies } M(c, Hu, z) = 1$$

$$\text{i.e.} \quad Hu = c \quad \text{---3.1.4}$$

From Equation 3.1.3 and Equation 3.1.4, we have

$$Hu = Gu = c \quad \text{--- 3.1.5}$$

By definition of weakly compatible, we have

$$\begin{aligned} & Hc = Gc \quad (\text{we assumed that } H \\ \text{and } G & \text{ are weakly compatible} ) \\ & \text{---3.1.6} \end{aligned}$$

Now fix that  $c$  is a fixed point of  $H$ . Conversely taking  $Hc \neq c$ .

Equations 3.1.5, 3.1.1, with 3.1.6 and using properties of  $\phi$ , we get

$$\begin{aligned} & \phi(M(Hc, c, z)) = \phi(M(Hc, Hu, z)) \\ & \leq \phi \{ \alpha_1(z) \frac{M(Gc, Hc, z)M(Gu, Hu, z)}{M(Gc, Gu, z)} \\ & \quad + \alpha_2(z) \frac{M(Gc, Hu, z)M(Hu, Hc, z)}{M(Gc, Gu, z)} \\ & \quad + \alpha_3(z) M(Hc, Gc, z) + \alpha_4(z) M(Gu, Hu, z) \\ & \quad + \alpha_5(z) M(Gc, Gu, z) + \alpha_6(z) M(Hc, Gu, 2z) \} \\ & \leq \phi \{ \alpha_1(z) \frac{M(Hc, Hc, z)M(c, c, z)}{M(Hc, c, z)} \\ & \quad + \alpha_2(z) \frac{M(Hc, cu, z)M(c, Hc, z)}{M(Hc, c, z)} + \alpha_3(z) M(Hc, Hc, z) \\ & \quad + \alpha_4(z) M(c, c, z) + \alpha_5(z)M(Hc, c, z) \\ & \quad + \alpha_6(z) M(Hc, c, 2z) \} \end{aligned}$$

Or

$$\phi(M(Hc, c, z)) \leq \phi ( \alpha_1(z) \frac{\phi(1).\phi(1)}{M(Hc, c, z)} )$$

$$\begin{aligned} & + \phi ( \alpha_2(z) M(Hc, c, z) ) + \alpha_3(z)\phi(1) \\ & + \alpha_4(z) \phi(1) + \phi ( \alpha_5(z) M(Hc, c, z) ) \\ & + \phi(\alpha_6(z) M(Hc, c, 2z) ) \end{aligned}$$

Or

$$(1 - \alpha_2(z) - \alpha_6(z)) \phi(M(Hc, c, z)) \leq \phi(\alpha_6(z) M(Hc, c, 2z)) < \phi(M(Hc, c, z)), \quad z > 0.$$

It's clear that  $Hc = c$ . Thus,

$$\begin{aligned} & Hc = c = Gc \text{ i.e. } c \text{ is a common fixed point of } H \\ \text{and } G & \\ & \text{---3.1.7} \end{aligned}$$

To establishment of Uniqueness taking  $\omega \in W$  is one more common fixed point of  $H$  and  $G$  such that

$$\begin{aligned} & H\omega = G\omega = \omega \text{ and } \omega \neq c \\ & \text{---3.1.8} \end{aligned}$$

Then by Equations 3.17, 3.1.8, with Equation 3.1.1 and properties of  $\phi$ , we have

$$\begin{aligned} & \phi(M(c, \omega, z)) = \phi(M(Hc, H\omega, z)) \\ & \leq \phi \{ \alpha_1(z) \frac{M(Gc, Hc, z)M(G\omega, H\omega, z)}{M(Gc, G\omega, z)} \\ & \quad + \alpha_2(z) \frac{M(Gc, H\omega, z)M(G\omega, Hc, z)}{M(Gc, G\omega, z)} + \alpha_3(z) M(Hc, Gc, z) \\ & \quad + \alpha_4(z) M(G\omega, H\omega, z) + \alpha_5(z) M(Gc, G\omega, z) \\ & \quad + \alpha_6(z) M(Hc, G\omega, 2z) \} \\ & \leq \phi \{ \alpha_1(z) \frac{M(c, c, z)M(\omega, \omega, z)}{M(c, \omega, z)} \\ & \quad + \alpha_2(z) \frac{M(c, \omega, z)M(\omega, c, z)}{M(c, \omega, z)} + \alpha_3(z) M(c, c, z) \\ & \quad + \alpha_4(z)M(\omega, \omega, z) + \alpha_5(z) M(c, \omega, z) \\ & \quad + \alpha_6(z) M(c, \omega, 2z) \} \end{aligned}$$

Or

$$(1 - \alpha_2(z) - \alpha_5(z)) \phi(M(c, \omega, z)) \leq$$

$$\alpha_6(z) \phi(M(c, \omega, 2z)) < \phi(M(c, \omega, z)), z > 0.$$

it's clear that  $c$  is a unique common fixed point of  $H$  and  $G$ .

## CONCLUSION

In present going-over by using two self mappings for re-inspection carried out and built a new version over famed theory with felt that it may further looking for more than two mappings with different spaces.

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