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Dynamic Gravity and International Inequality

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Abstract

After modifying and extending the Olivero and Yotov (2012) dynamic gravity model, I derive key novel theoretical findings: the *balance condition* and the *barrier-flow dynamic gravity relationship*. The balance condition shows that growth of a country's market share and trade ease puts downward pressure on the market share and trade ease of other countries, highlighting the link between international inequality and trade frictions. The barrier-flow dynamic gravity relationship shows that relative trade flows growth rate is inversely proportional to relative trade barriers growth rate.

Keywords: Trade; Gravity; Inequality; Dynamic

1. Introduction

The foundation of gravity is rooted in the field of physics. The famous Newton's Law of Gravity relates the force (F) between two masses, to their product (m_1m_2), the distance between them (d), and a gravitational constant (G):

$$F = Gm_1m_2/d^2 \quad (1)$$

The application of gravity to social sciences can be traced back to Carey (1865), with an analysis of migration flows. The prominence of the gravity equation within "social physics" is largely due to Stewart (1948), with an analysis of demographic gravitation. Subsequent works made significant contributions in applying the gravity equation, including Anderson (1979) who described frictionless gravity and a demand-side structural gravity model. The focus in this paper will be on demand-side structural gravity. In the next section I review some important works in the theory of gravity. Then I build on the model by Olivero and Yotov (2012), by expressing the *balance condition* and the *barrier-flow dynamic gravity relationship*. In this paper I contribute to the existing demand-side structural gravity theory by expressing dynamic gravity and relating it to international inequality.

2. Background

Anderson (1979) lays out the theoretical foundations, which are also summarized in Anderson (2011). In a frictionless and homogeneous world, each good has the same price everywhere, and agents purchase goods in the same proportions everywhere. Consumers in country j consume country i's goods (X_{ij}), which as a fraction of country j's expenditure (E_j) equals the ratio of country i's income (Y_i) to the world income (Y). The gravity relationship for aggregated goods can be expressed in terms of country j's expenditure share ($b_j = E_j/Y$) and country i's market share ($s_i = Y_i/Y$):

$$X_{ij} = E_jY_i/Y = b_js_iY \quad (2)$$

The above can also be expressed in terms of disaggregated goods, indexed by k:

$$X_{ij}^k = E_j^kY_i^k/Y^k = b_j^ks_i^kY^k \quad (3)$$

The relation in (2) can also be expressed in terms of bilateral shares, with $s_i^{ij} = Y_i/(Y_i+Y_j)$ representing the share of i in the joint income of i and j. Along with balanced aggregate trade ($b_j = s_j$), that would imply:

$$X_{ij} = s_i^{ij} s_j^{ij} \frac{(Y_i+Y_j)^2}{Y} \tag{4}$$

A measure for world openness (WO) capturing the relative number of exports across all countries is given by:

$$WO = \sum_j \sum_{i \neq j} X_{ij}/Y = \sum_j \sum_{i \neq j} b_j s_i = \sum_j b_j (1-s_j) = 1 - \sum_j b_j s_j \tag{5}$$

Given the theoretical foundations of frictionless gravity, it is natural to then think about and model the frictions that occur in trade gravity.

Considering demand-side structural gravity, Anderson (1979) uses Cobb-Douglas preferences, as well as constant elasticity of substitution (CES) preferences, to derive a theoretical gravity foundation. Using both preferences, Deardorff (1998) presents a Heckscher-Ohlin model with complete specialization. Anderson and van Wincoop (2003), using CES preferences, present a relationship between gravity and multilateral resistance. Namely, consumers in country j maximize their utility made up of their consumption on goods from country i, denoted by C_{ij}:

$$\left(\sum_i \beta_i^{(1-\sigma)/\sigma} C_{ij}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{6}$$

subject to:

$$\sum_i X_{ij} = \sum_i p_i \tau_{ij} C_{ij} = Y_j \tag{7}$$

Note that β_i is the taste parameter, $\sigma > 1$ is the elasticity of substitution, p_i is the exporter's supply price, while $\tau_{ij} \geq 1$ represents the trade cost factor between i and j (which is assumed to be borne by the exporter). The above maximization gives the following:

$$\frac{X_{ij}}{E_j} = \frac{X_{ij}}{Y_j} = \frac{(\beta_i p_i \tau_{ij})^{1-\sigma}}{\sum_i (\beta_i p_i \tau_{ij})^{1-\sigma}} = \left(\frac{\beta_i p_i \tau_{ij}}{P_j} \right)^{1-\sigma} \tag{8}$$

Note that $P_j^{1-\sigma} = \sum_i (\beta_i p_i \tau_{ij})^{1-\sigma}$, where P_j is the inward multilateral resistance (the buyers' incidence of trade costs). Furthermore, market clearance ($Y_i = \sum_j X_{ij}$) yields:

$$(\beta_i p_i)^{1-\sigma} = Y_i / \sum_j Y_j (\tau_{ij}/P_j)^{1-\sigma} \tag{9}$$

Substituting the above into the expression for X_{ij} gives:

$$X_{ij} = \frac{Y_i Y_j (\tau_{ij}/P_j)^{1-\sigma}}{\sum_j Y_j (\tau_{ij}/P_j)^{1-\sigma}} \tag{10}$$

Relating the above to the market share ($s_j = Y_j/Y$) yields a key expression:

$$X_{ij} = \frac{Y_i Y_j (\tau_{ij}/P_j)^{1-\sigma}}{Y \sum_j s_j (\tau_{ij}/P_j)^{1-\sigma}} = \frac{Y_i Y_j (\tau_{ij}/P_j)^{1-\sigma}}{Y \Pi_i^{1-\sigma}} = \left(\frac{\tau_{ij}}{\Pi_i P_j} \right)^{1-\sigma} \frac{Y_i Y_j}{Y} \tag{11}$$

Note that $\Pi_i^{1-\sigma} = \sum_j s_j (\tau_{ij}/P_j)^{1-\sigma}$, where Π_i is the outward multilateral resistance (the sellers' incidence of trade costs). Setting $p_i = 1$ in (8), along with (11) gives:

$$\beta_i^{1-\sigma} = s_i / \Pi_i^{1-\sigma} \tag{12}$$

Therefore, the inward multilateral resistance, referred to as the consumer price index (P_j), can be expressed in terms of the outward multilateral resistance (Π_i):

$$P_j^{1-\sigma} = \sum_i s_i (\tau_{ij}/\Pi_i)^{1-\sigma} \tag{13}$$

With a huge assumption of symmetric trade barriers ($\tau_{ij} = \tau_{ji}$), then $\Pi_i = P_i$, and the expression for X_{ij} from (11) becomes:

$$X_{ij} = \left(\frac{\tau_{ij}}{P_i P_j} \right)^{1-\sigma} \frac{Y_i Y_j}{Y} \tag{14}$$

The three implications given by Anderson and van Wincoop (2003) are that for a uniform rise in trade barriers between all countries and assuming that each country is frictionless: (a) trade barriers reduce size-adjusted trade between large countries more than between small countries, (b) trade barriers raise size-adjusted trade within small countries more than within large countries, and (c) trade barriers raise the ratio of size-adjusted trade within country 1 relative to size-adjusted trade between countries 1 and 2 by more the smaller is country 1 and the larger is country 2.

The expression in (11) also gives the Constructed Trade Bias (ratio of predicted trade to predicted frictionless trade): $(\tau_{ij}/\Pi_i P_j)^{1-\sigma}$. Constructed Home Bias is thus: $(\tau_{ii}/\Pi_i P_i)^{1-\sigma}$. Using (11) for intranational trade, Novy (2013) expresses the product of the multilateral resistance terms:

$$\Pi_i P_i = \left(\frac{X_{ii}/Y_i}{Y_i/Y}\right)^{\frac{1}{\sigma-1}} \tau_{ii} \tag{15}$$

Expressing the above for country j too and combining it with (11) and (15) gives:

$$\frac{\tau_{ij}\tau_{ji}}{\tau_{ii}\tau_{jj}} = \left(\frac{X_{ij}X_{ji}}{X_{ii}X_{jj}}\right)^{\frac{1}{1-\sigma}} \tag{16}$$

This shows an inverse (since $\sigma > 1$) relationship between relative trade barriers and relative trade flows. (Later I extend that relationship to dynamic gravity.) It also implies that higher heterogeneity (lower σ) is associated with higher relative trade barriers, for given relative trade flows. It should also be noted that with balanced aggregate trade ($b_i = s_i$) expression (15) implies that $\Pi_i P_i = \tau_{ii}$, thus making the Constructed Home Bias equal to 1.

Adding dynamics to gravity theory is useful, as trade is dynamic by nature. It also allows for estimating gravity using panel data. It can address the persistence in trade flows and trade barriers. Also, specific to the above Anderson and van Wincoop (2003) model, it can explore the dynamics of the multilateral resistance terms. Olivero and Yotov (2012) develop the theoretical foundations for a dynamic gravity model. They develop a theoretical general equilibrium model by incorporating the static endowment-based model of Anderson and van Wincoop (2003), and the dynamic two-country macroeconomic model of Backus, Kehoe, and Kydland (1994). Their resulting model is in discrete time, with a CES utility function and a Cobb-Douglas production function. The model's labor stock is assumed to be equal to 1 (across countries and time), while the capital stock follows a standard law of motion. They decompose trade flows and trade barriers into static and dynamic components, elegantly highlighting their effects on the economy.

The aim of this paper is to contribute to gravity theory in several ways. First, the paper modifies the Olivero and Yotov (2012) dynamic gravity model to include a dynamic labor stock, since population growth can affect income and inequality. Assuming a constant labor stock across all countries does not capture the varying dynamics of labor's effect on income per capita across countries. Differences in population growth rates can be an important source of inequality between countries. Then, the paper derives novel gravity expressions for income and trade flows, along with the paper's key relationships: the *balance condition* and the *barrier-flow dynamic gravity relationship*. The purpose of these expressions lies in providing a relationship between trade flows, trade barriers, and inequality, with mechanisms founded on the dynamic gravity model, rather than relying on the static gravity model. Ultimately, the paper describes trade flows and trade barriers, advancing their relationship to growth rates and to the international income per capita inequality, highlighting the various components of the inequality. The paper aims to provide a useful theoretical contribution for analyzing trade flows, trade barriers, and inequality, in the context of dynamic gravity.

3. Model

3.1 Setup

Let the consumers in country j choose their aggregate consumption (C_{jt}) out of their aggregate income (Y_{jt}). Their savings make up their aggregate investment (I_{jt}). They maximize their lifetime welfare (W_j), which depends on a time discount rate ($\rho > 0$), and constant elasticity of substitution (CES) functions of consumption (C_{jt}) and investment (I_{jt}). The CES functions differ from those in the Olivero and Yotov (2012) dynamic model, as I set the taste parameter (β_i) equal to 1, like the Bergstrand (1985) model:

$$W_j = \sum_t^{\infty}(1+\rho)^{-t}(C_{jt}) \tag{17}$$

$$C_{jt} = \left(\sum_i C_{ijt}^{\gamma}\right)^{1/\gamma} \tag{18}$$

$$I_{jt} = \left(\sum_i I_{ijt}^{\gamma}\right)^{1/\gamma} \tag{19}$$

With $0 < \gamma < 1$, the elasticity of substitution is $\sigma = 1/(1 - \gamma) > 1$. Furthermore, let X_{ijt} denote country j's nominal spending on goods from country i, with P_{ijt} being the price of consumption goods, and Z_{ijt} being the price of investment goods:

$$X_{ijt} = P_{ijt}C_{ijt} + Z_{ijt}I_{ijt} \tag{20}$$

Note that $P_{ijt} = p_{it}\tau_{ijt}$ and $Z_{ijt} = z_{it}\tau_{ijt}$, where p_{it} and z_{it} denote the exporter's factory-gate prices, while $\tau_{ijt} \geq 1$ denotes trade barriers for shipments from i to j, which can be asymmetric ($\tau_{ijt} \neq \tau_{jit}$). For home shipments, it is assumed that $\tau_{iit} = \tau_{jjt} = 1$. In the Olivero and Yotov (2012) model, the consumption goods and investment goods are priced the same ($p_{it} = z_{it}$). For now, I can assume that as well. (Consumption goods and investment goods are priced differently in the Appendix.)

Consumer choices are bound by constraints. Country j's aggregate nominal spending on all goods equals aggregate income:

$$Y_{jt} = \sum_i X_{ijt} \tag{21}$$

The aggregate income follows a Cobb-Douglas production function. Labor (L_{jt}) and capital (K_{jt}) are used to produce a single product at price p_{jt} with $\alpha \in (0, 1)$:

$$Y_{jt} = p_{jt} L_{jt}^{1-\alpha} K_{jt}^{\alpha} \tag{22}$$

Population growth plays a role in the dynamics of trade and inequality. Unlike the Olivero and Yotov (2012) model, I do not assume that labor is fixed at 1, but rather that labor in country j grows at rate η_{jt}:

$$L_{jt} = (1+\eta_{jt}) L_{jt-1} \tag{23}$$

Finally, capital is the accumulation of investment and existing capital (depreciating at rate δ_j):

$$K_{jt} = I_{jt} + (1-\delta_j) K_{jt-1} \tag{24}$$

3.2 Optimality

To derive optimal conditions, consumers in country j maximize (17) subject to (18)-(24). This gives their nominal spending on goods from country i as the dynamic equivalent of Anderson and van Wincoop (2003), as in (8):

$$X_{ijt} = \left(\frac{P_{ijt}}{P_{jt}}\right)^{1-\sigma} Y_{jt} \tag{25}$$

The price index is $P_{jt} = [\sum_i P_{ijt}^{1-\sigma}]^{1/(1-\sigma)}$, while the product price is $P_{ijt} = p_{it}\tau_{ijt}$. Note also that the partial elasticity of relative imports with respect to variable trade barriers, given by $\partial \ln(X_{ijt}/X_{jjt})$, is $1-\sigma$, which Arkolakis, Costinot, and Rodríguez-Clare (2012) refer to as the trade elasticity. Furthermore, using market clearance ($\sum_j X_{ijt} = Y_{it}$) and world income ($Y_t = \sum_i Y_{it} = \sum_j Y_{jt}$), this becomes as in (11):

$$X_{ijt} = \left(\frac{\tau_{ijt}}{\Pi_{it} P_{jt}}\right)^{1-\sigma} \frac{Y_{it} Y_{jt}}{Y_t} \tag{26}$$

Note that $\Pi_{it}^{1-\sigma} = \sum_j (Y_{jt}/Y_t)(\tau_{ijt}/P_{jt})^{1-\sigma}$ and also $P_{jt}^{1-\sigma} = \sum_i (Y_{it}/Y_t)(\tau_{ijt}/\Pi_{it})^{1-\sigma}$. Next, I can combine the economy's constraints (22)-(24) to get:

$$\left(\frac{Y_{jt}}{p_{jt}}\right)^{\frac{1}{\alpha}} = L_{jt}^{\frac{1}{\alpha}} \frac{I_{jt}}{L_{jt}} + (1-\delta_j)(1+\eta_{jt})^{\frac{1-\alpha}{\alpha}} \left(\frac{Y_{jt-1}}{p_{jt-1}}\right)^{\frac{1}{\alpha}} \tag{27}$$

Olivero and Yotov (2012) define real income (output) as Y_{jt}/p_{jt}. Applying their definition would then define real income per capita as r_{jt} = Yjt/($p_{jt}L_{jt}$). Also, if investment per capita is defined as $i_{jt} = I_{jt}/L_{jt}$, then the above relationship can be expressed as: $r_{jt}^{1/\alpha} = i_{jt} + (1 - \delta_j)(1 + \eta_{jt})^{(\alpha-1)/\alpha} r_{jt-1}^{1/\alpha}$. While this may be convenient, defining real income as Y_{jt}/p_{jt} is problematic and inconsistent with the expressions in (20) and (21). Namely, consumers in country j consume goods from their own country and from other countries i, thus making the prices from countries i also relevant when measuring real income. Therefore, relating nominal income (Y_{jt}) to just the home price (p_{jt}) is inconsistent. To define real income, it would be more consistent to relate the nominal income to the price index (P_{jt}), which includes the prices of all the goods that country j consumers buy, as in Arkolakis et al. (2012). Furthermore, Olivero and Yotov (2012) use the ratio $I_{jt}/(Y_{jt}/p_{jt})$ in their analysis, which they define as the investment share of real output. In this paper's analysis, I can instead focus on the capital growth rate, which I can denote as $\kappa_{jt} = K_{jt}/K_{jt-1} - 1$.

With (23), (26) for Y_{jt-1}, and investment per capita ($i_{jt} = I_{jt}/L_{jt}$), then (27) gives:

$$Y_{jt} = \left[i_{jt}(p_{jt}L_{jt})^{\frac{1}{\alpha}} + (1-\delta_j)(1+\eta_{jt})^{\frac{1-\alpha}{\alpha}} X_{ijt-1}^{\frac{1}{\alpha}} \left(\frac{\tau_{ijt-1}^{\sigma-1} Y_{t-1} p_{jt}}{(\Pi_{it-1}P_{jt-1})^{\sigma-1} Y_{it-1} p_{jt-1}}\right)^{\frac{1}{\alpha}} \right]^{\alpha} \tag{28}$$

Then along with (22) and the capital growth rate (κ_{jt}), the above becomes:

$$Y_{jt} = (1+\eta_{jt})^{1-\alpha}(1+\kappa_{jt})^{\alpha} X_{ijt-1} \left(\frac{\tau_{ijt-1}}{\Pi_{it-1}P_{jt-1}}\right)^{\sigma-1} \left(\frac{Y_{t-1}}{Y_{it-1}}\right) \left(\frac{p_{jt}}{p_{jt-1}}\right) \tag{29}$$

Using market share ($s_{it} = Y_{it}/Y_t$) and home price inflation rate ($\pi_{jt} = p_{jt}/p_{jt-1} - 1$), country j's income becomes:

$$Y_{jt} = (1+\pi_{jt})(1+\eta_{jt})^{1-\alpha}(1+\kappa_{jt})^{\alpha} \left(\frac{X_{ijt-1}}{s_{it-1}}\right) \left(\frac{\tau_{ijt-1}}{\Pi_{it-1}P_{jt-1}}\right)^{\sigma-1} \tag{30}$$

Then using (23) country j's income per capita ($YPC_{jt} = Y_{jt}/L_{jt}$) can be expressed as:

$$YPC_{jt} = (1+\pi_{jt}) \left(\frac{1+\kappa_{jt}}{1+\eta_{jt}}\right)^{\alpha} \left(\frac{X_{ijt-1}}{s_{it-1}L_{jt-1}}\right) \left(\frac{\tau_{ijt-1}}{\Pi_{it-1}P_{jt-1}}\right)^{\sigma-1} \tag{31}$$

Plugging (30) into (26) gives the paper's *structural dynamic gravity equation*:

$$X_{ijt} = X_{ijt-1}(1+\pi_{jt})(1+\eta_{jt})^{1-\alpha}(1+\kappa_{jt})^{\alpha}\left(\frac{s_{it}}{s_{it-1}}\right)\left(\frac{\tau_{ijt}\Pi_{it-1}P_{jt-1}}{\tau_{ijt-1}\Pi_{it}P_{jt}}\right)^{1-\sigma} \quad (32)$$

The above structural dynamic gravity equation differs from the structural dynamic gravity equation of Olivero and Yotov (2012) by including population growth, and by relating current trade flows to lagged trade flows and growth rates. Specifically, it relates trade flows from country i to country j at time t to: its lag (X_{ijt-1}), trade barriers (τ_{ijt}, τ_{ijt-1}), multilateral resistance terms (Π_{it}, P_{jt}, Π_{it-1}, P_{jt-1}), exporter's market shares (s_{it}, s_{it-1}), importer's capital growth rate (κ_{jt}), labor growth rate (η_{jt}), and home price inflation rate (π_{jt}), for given $\alpha \in (0, 1)$ and $\sigma > 1$.

Looking at (32) several observations can be made. A rise in the lagged trade flows has a positive effect on the current trade flows ($\partial X_{ijt}/\partial X_{ijt-1} > 0$), thus implying that habits matter. Countries that traded with each other in the previous time period are likely to continue to trade with each other. Olivero and Yotov (2012) refer to this as the trade persistence effect. Furthermore, trade barriers have static and dynamic effects. The static effect on trade flows is negative ($\partial X_{ijt}/\partial \tau_{ijt} < 0$), while the dynamic effect is positive ($\partial X_{ijt}/\partial \tau_{ijt-1} > 0$). Olivero and Yotov (2012) refer to the positive dynamic effect as the protection persistence effect. Namely, importer's protection can lead to higher capital formation and production, and thus create pressure for higher imports in the future. Exporter's market share also has static and dynamic effects. The static effect of the market share on trade flows is positive ($\partial X_{ijt}/\partial s_{it} > 0$), while the dynamic effect is negative ($\partial X_{ijt}/\partial s_{it-1} < 0$). The negative dynamic effect implies that if country i becomes bigger (by its market share), then there is pressure for country j to import less from country i in the future.

In terms of growth rates, importer's capital growth rate has a positive effect on trade flows ($\partial X_{ijt}/\partial \kappa_{jt} > 0$), which reflects low depreciation of capital relative to the new capital (investment), thus fueling higher income and consumption. Also, higher importer's labor growth rate is associated with higher trade flows ($\partial X_{ijt}/\partial \eta_{jt} > 0$), as higher populations consume more. This is important since population growth affects trade flows, which can affect the inequality between countries. Higher importer's home price inflation rate is also associated with higher trade flows ($\partial X_{ijt}/\partial \pi_{jt} > 0$). This effect of the home price can further be broken down into a positive static effect ($\partial X_{ijt}/\partial p_{jt} > 0$), and a negative dynamic effect ($\partial X_{ijt}/\partial p_{jt-1} < 0$). Thus, while home price rising increases current trade flows, it creates pressure on future trade flows to decrease.

The $(\tau_{ijt}/\Pi_{it}P_{jt})^{1-\sigma}$ term in (32) represents the inverse of the trade frictions for shipments from i to j (where a higher value represents lower frictions, and thus easier trade). The growth rate of trade ease is $\xi_{ijt} = (\tau_{ijt}/\Pi_{it}P_{jt})^{1-\sigma}(\tau_{ijt-1}/\Pi_{it-1}P_{jt-1})^{\sigma-1} - 1$. Also, exporter's market share growth rate is $\mu_{it} = s_{it}/s_{it-1} - 1$, and the growth rate of the trade flows is $\chi_{ijt} = X_{ijt}/X_{ijt-1} - 1$. Then (32) can be expressed in terms of growth rates:

$$(1+\chi_{ijt}) = (1+\xi_{ijt})(1+\mu_{it})(1+\pi_{jt})(1+\eta_{jt})^{1-\alpha}(1+\kappa_{jt})^{\alpha} \quad (33)$$

Hence, the growth rate of trade flows is a function of the growth rates of exporter's trade ease and market share, as well as importer's home price, population, and capital.

The expression for country j's income from (30) can be expressed using the aggregation of X_{ijt} from (32) across countries i using (21). This results in:

$$\left(\frac{X_{ijt-1}}{s_{it-1}}\right)\left(\frac{\tau_{ijt-1}}{\Pi_{it-1}P_{jt-1}}\right)^{\sigma-1} = \sum_i X_{ijt-1}\left(\frac{s_{it}}{s_{it-1}}\right)\left(\frac{\tau_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma}\left(\frac{\tau_{ijt-1}}{\Pi_{it-1}P_{jt-1}}\right)^{\sigma-1} \quad (34)$$

Therefore, country j's income can be expressed as:

$$Y_{jt} = (1+\pi_{jt})(1+\eta_{jt})^{1-\alpha}(1+\kappa_{jt})^{\alpha}\sum_i X_{ijt-1}\left(\frac{s_{it}}{s_{it-1}}\right)\left(\frac{\tau_{ijt}\Pi_{it-1}P_{jt-1}}{\tau_{ijt-1}\Pi_{it}P_{jt}}\right)^{1-\sigma} \quad (35)$$

The above can also be expressed in terms of growth rates:

$$Y_{jt} = (1+\pi_{jt})(1+\eta_{jt})^{1-\alpha}(1+\kappa_{jt})^{\alpha}\sum_i X_{ijt-1}(1+\mu_{it})(1+\xi_{ijt}) \quad (36)$$

Using (23) and (36) gives country j's income per capita, which is also in (31):

$$YPC_{jt} = \left(\frac{1+\pi_{jt}}{L_{jt-1}}\right)\left(\frac{1+\kappa_{jt}}{1+\eta_{jt}}\right)^{\alpha}\sum_i X_{ijt-1}(1+\mu_{it})(1+\xi_{ijt}) \quad (37)$$

It follows from (22) that the summation component in (36) and (37) is equal to the lag of country j's income:

$$Y_{jt-1} = \sum_i X_{ijt-1}(1+\mu_{it})(1+\xi_{ijt}) \quad (38)$$

Therefore, income per capita can be expressed as:

$$YPC_{jt} = \sum_i X_{ijt}(1+\mu_{it+1})(1+\xi_{ijt+1})/L_{jt} \quad (39)$$

Expanding (38) and aggregating gives:

$$Y_{jt-1} = \sum_i X_{ijt-1}(1+\xi_{ijt}+\mu_{it}+\xi_{ijt}\mu_{it}) \quad (40)$$

$$Y_{jt-1} = \sum_i X_{ijt-1} + \sum_i X_{ijt-1}(\xi_{ijt}+\mu_{it}+\xi_{ijt}\mu_{it}) \quad (41)$$

Since $\sum_i X_{ijt-1} = Y_{jt-1}$ by (21), then (41) simplifies to:

$$0 = \sum_i X_{ijt-1}(\xi_{ijt} + \mu_{it} + \xi_{ijt}\mu_{it}) \quad (42)$$

This is the paper's *balance condition*, such that if some country's market share is rising ($\mu_{it} > 0$) and its trade ease is rising ($\xi_{ijt} > 0$), then another country's market share and trade ease are falling. For instance, if some country is growing and it can export easier, then it is harder for another country to grow and export. This is an important relationship which highlights the interdependence of countries, and how the inequality of market shares (and therefore income) is related to the inequality of trade barriers. Thus, if the dynamics of trade barriers differ between countries, then the dynamics of market shares will react. Growth of a country's market share and trade ease puts downward pressure on the market share and trade ease of other countries. If we imagine all countries starting with symmetric trade barriers and symmetric market shares, and then trade barriers become asymmetric (causing trade ease growth rates to be different between countries), then market share growth rates will react and differ between countries, creating international inequality.

The analysis can further model the dynamics of trade flows and their relationship to trade barriers. The structural dynamic gravity equation in (32) can be used to find intranational trade flows (X_{iit} and X_{jjt}). Then, using the X_{iit} and X_{jjt} expressions, the intranational multilateral resistance terms ($\Pi_{it}P_{it}$ and $\Pi_{jt}P_{jt}$) can be plugged into the product of X_{ijt} and X_{jit}. This isolates the product of the trade barriers ($\tau_{ijt}\tau_{jit}$). Then, I can form the ratio of the product of the international trade barriers to the product of the intranational trade barriers, and relate their ratio to the ratio of the product of the international trade flows to the product of the intranational trade flows. The dynamic result reveals the paper's *barrier-flow dynamic gravity relationship*:

$$\frac{\left(\frac{X_{ijt}X_{jit}}{X_{iit}X_{jjt}}\right)}{\left(\frac{X_{ijt-1}X_{jit-1}}{X_{iit-1}X_{jjt-1}}\right)} = \frac{\left(\frac{\tau_{ijt}\tau_{jit}}{\tau_{iit}\tau_{jjt}}\right)^{1-\sigma}}{\left(\frac{\tau_{ijt-1}\tau_{jit-1}}{\tau_{iit-1}\tau_{jjt-1}}\right)^{1-\sigma}} \quad (43)$$

The term ($X_{ijt}X_{jit}/X_{iit}X_{jjt}$) represents the international trade flows relative to intranational trade flows, while the term ($\tau_{ijt}\tau_{jit}/\tau_{iit}\tau_{jjt}$) represents the international trade barriers relative to intranational trade barriers. The ratio of each term and its lag reflect the growth of that term. Thus, given the elasticity of substitution ($\sigma > 1$), the relationship in (43) shows that the relative trade flows growth rate is inversely proportional to the relative trade barriers growth rate. This dynamic result advances the static relationship in (16), which suggests an inverse relationship between relative trade flows and relative trade barriers. Through dynamic gravity, the analysis here shows that the inverse relationship applies to their growth rates.

3.3 Inequality

The balance condition in (42) shows an underlying relationship between inequality and trade frictions. For expressing income per capita inequality, I can first use (31) for YPC_{it} and YPC_{jt} and then denote their log difference as $Q_{ijt}^{YPC} = \ln(YPC_{it}/YPC_{jt})$:

$$\begin{aligned} Q_{ijt}^{YPC} = {} & ln\left(\frac{1+\pi_{it}}{1+\pi_{jt}}\right) + \alpha ln\left(\frac{1+\kappa_{it}}{1+\kappa_{jt}}\right) + \alpha ln\left(\frac{1+\eta_{jt}}{1+\eta_{it}}\right) + ln\left(\frac{X_{jit-1}}{X_{ijt-1}}\right) \\ & + ln\left(\frac{s_{it-1}}{s_{jt-1}}\right) + ln\left(\frac{L_{jt-1}}{L_{it-1}}\right) + (\sigma-1)ln\left(\frac{\tau_{jit-1}}{\tau_{ijt-1}}\right) \\ & + (\sigma-1)ln\left(\frac{\Pi_{it-1}}{\Pi_{jt-1}}\right) + (\sigma-1)ln\left(\frac{P_{jt-1}}{P_{it-1}}\right) \end{aligned} \quad (44)$$

Dividing YPC_{it} by P_{it} and YPC_{jt} by P_{jt} would give real income per capita values. Summing the squares of the log difference in (44) gives a measure of international income per capita inequality, denoted as Q_t^{YPC}:

$$Q_t^{YPC} = \sum_i \sum_j (Q_{ijt}^{YPC})^2 \quad (45)$$

The inequality of trade barriers between trading partners can be expressed by using (32) to isolate τ_{ijt} and τ_{jit}, and then taking their log difference, denoted as $Q_{ijt}^{TB} = \ln(\tau_{ijt}/\tau_{jit})$:

$$\begin{aligned} Q_{ijt}^{TB} = {} & \frac{1}{\sigma-1}ln\left(\frac{1+\chi_{jit}}{1+\chi_{ijt}}\right) + \frac{1}{\sigma-1}ln\left(\frac{1+\pi_{jt}}{1+\pi_{it}}\right) + \frac{1-\alpha}{\sigma-1}ln\left(\frac{1+\eta_{jt}}{1+\eta_{it}}\right) \\ & + \frac{\alpha}{\sigma-1}ln\left(\frac{1+\kappa_{jt}}{1+\kappa_{it}}\right) + \frac{1}{\sigma-1}ln\left(\frac{1+\mu_{it}}{1+\mu_{jt}}\right) + ln\left(\frac{\Pi_{it}P_{jt}}{\Pi_{jt}P_{it}}\right) \\ & + ln\left(\frac{\tau_{ijt-1}}{\tau_{jit-1}}\right) + ln\left(\frac{\Pi_{jt-1}}{\Pi_{it-1}}\right) + ln\left(\frac{P_{it-1}}{P_{jt-1}}\right) \end{aligned} \quad (46)$$

Trade barriers are sometimes conveniently assumed to be symmetric ($\tau_{ijt} = \tau_{jit}$), thus making the trade barriers ratio equal to 1, and their asymmetry (inequality) expressed by Q_{ijt}^{TB} equal to 0. That is not a reasonable assumption, as clearly there are many reasons for the trade barriers to be asymmetric. Furthermore, the asymmetry of trade barriers is related to the asymmetries of trade flows and market shares, as inferred from (42), and the asymmetry of income per capita values, as inferred from (44). As the analysis showed earlier, a higher market share and trade ease in one country puts downward pressure on the market share and trade ease of other countries. The asymmetry of trade barriers affects the asymmetry of market shares and the international income per capita inequality.

For future potential research, this interdependence and its effect on inequality can be empirically modeled to estimate the asymmetry of trade barriers. After empirical investigation, it can lead to further questions on the nature of asymmetric trade barriers, the role of trade agreements, and the implications for international trade policy.

4. Conclusion

In this paper I extended a dynamic gravity model introduced by Olivero and Yotov (2012). Including a dynamic labor stock, I derived optimal trade flows and their growth rates, relating them to income components and trade frictions. As a result, I derived the *balance condition* that relates trade flows to the growth rates of trade ease and market share. This novel gravity condition highlights the interdependence of countries, and how the inequality of market shares is related to the inequality of trade barriers. Growth of a country's market share and trade ease puts downward pressure on the market share and trade ease of other countries. Furthermore, in the *barrier-flow dynamic gravity relationship* I expressed relative trade flows and relative trade barriers, quantifying the ratios of the international to the intranational. It follows that the relative trade flows growth rate is inversely proportional to the relative trade barriers growth rate. Overall, the paper introduced international inequality within the context of dynamic gravity theory, bringing attention to the dynamics and asymmetries of trade barriers and incomes.

Appendix

Here I assume that consumption goods and investment goods are priced differently ($p_{it} \neq z_{it}$), to see the dynamics between the two types of consumer behavior (spending and saving). I augment the utility function (17) to include investment goods as well: $W = \sum_t (1+\rho)^{-t}(C_{jt}+I_{jt})$. When consumers in country j maximize the utility subject to (18)-(24), their nominal spending on goods from country i becomes:

$$X_{ijt} = \frac{\left[(\sum_i C_{ijt}^{\gamma})^{\frac{1-\gamma}{\gamma}} C_{ijt}^{\gamma} + (\sum_i I_{ijt}^{\gamma})^{\frac{1-\gamma}{\gamma}} I_{ijt}^{\gamma}\right]}{(\sum_i C_{ijt}^{\gamma})^{\frac{1}{\gamma}} + (\sum_i I_{ijt}^{\gamma})^{\frac{1}{\gamma}}} Y_{jt} \qquad (47)$$

I can use (18) and (19) to express the above as:

$$X_{ijt} = \left(\frac{C_{jt}}{C_{jt}+I_{jt}}\right)\left(\frac{C_{ijt}}{C_{jt}}\right)^{\gamma} Y_{jt} + \left(\frac{I_{jt}}{C_{jt}+I_{jt}}\right)\left(\frac{I_{ijt}}{I_{jt}}\right)^{\gamma} Y_{jt} \qquad (48)$$

Note that nominal spending is a function of fractions. Specifically, the intensity of consumption goods bought by consumers j relative to all the goods bought by consumers j can be labeled as: $\Omega_{jt} = C_{jt}/(C_{jt}+I_{jt})$. Then, $1 - \Omega_{jt} = I_{jt}/(C_{jt}+I_{jt})$. Hence, the above becomes:

$$X_{ijt} = (\Omega_{jt})\left(\frac{C_{ijt}}{C_{jt}}\right)^{\gamma} Y_{jt} + (1-\Omega_{jt})\left(\frac{I_{ijt}}{I_{jt}}\right)^{\gamma} Y_{jt} \qquad (49)$$

Note then that the nominal spending on consumption goods ($\sum_i P_{ijt}C_{ijt}$) is thus $(\Omega_{jt})Y_{jt}$, and the nominal spending on investment goods ($\sum_i Z_{ijt}I_{ijt}$) is thus $(1-\Omega_{jt})Y_{jt}$. Using the first-order conditions, along with (18)-(21), and denoting country j's price indices as $P_{jt} = [\sum_i P_{ijt}^{1-\sigma}]^{1/(1-\sigma)}$ and $Z_{jt} = [\sum_i Z_{ijt}^{1-\sigma}]^{1/(1-\sigma)}$, the above can be expressed as:

$$X_{ijt} = (\Omega_{jt})\left(\frac{P_{ijt}}{P_{jt}}\right)^{1-\sigma} Y_{jt} + (1-\Omega_{jt})\left(\frac{Z_{ijt}}{Z_{jt}}\right)^{1-\sigma} Y_{jt} \qquad (50)$$

With $\Pi_{it}^{1-\sigma} = \sum_j (Y_{jt}/Y_t)(\tau_{ijt}/P_{jt})^{1-\sigma}$, $P_{jt}^{1-\sigma} = \sum_i (Y_{it}/Y_t)(\tau_{ijt}/\Pi_{it})^{1-\sigma}$, $\Phi_{it}^{1-\sigma} = \sum_j (Y_{jt}/Y_t)(\tau_{ijt}/Z_{jt})^{1-\sigma}$, and $Z_{jt}^{1-\sigma} = \sum_i (Y_{it}/Y_t)(\tau_{ijt}/\Phi_{it})^{1-\sigma}$, the above becomes:

$$X_{ijt} = (\Omega_{jt})\left(\frac{\tau_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma}\frac{Y_{it}Y_{jt}}{Y_t} + (1-\Omega_{jt})\left(\frac{\tau_{ijt}}{\Phi_{it}Z_{jt}}\right)^{1-\sigma}\frac{Y_{it}Y_{jt}}{Y_t} \qquad (51)$$

Or more compactly it can be expressed as:

$$X_{ijt} = \left[(\Omega_{jt})\left(\frac{\tau_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma} + (1-\Omega_{jt})\left(\frac{\tau_{ijt}}{\Phi_{it}Z_{jt}}\right)^{1-\sigma}\right]\frac{Y_{it}Y_{jt}}{Y_t} \qquad (52)$$

The expression in the square brackets can be denoted as Λ_{ijt}. Note that if $p_{it} = z_{it}$, then (50) becomes (25) and (52) becomes (26). With the economy's constraints (22)-(24) giving expression (27) as before (whether $p_{it} = z_{it}$ or $p_{it} \neq z_{it}$), then with the modified expression for X_{ijt}, I can substitute for Y_{jt-1} in (27) using (52) to get:

$$Y_{jt} = \left[i_{jt}(p_{jt}L_{jt})^{\frac{1}{\alpha}} + \frac{(1-\delta_j)(1+\eta_{jt})^{\frac{1-\alpha}{\alpha}} X_{ijt-1}^{\frac{1}{\alpha}}\left(\frac{Y_{t-1}}{Y_{it-1}}\right)^{\frac{1}{\alpha}}\left(\frac{p_{jt}}{p_{jt-1}}\right)^{\frac{1}{\alpha}}}{\left[(\Omega_{jt-1})\left(\frac{\tau_{ijt-1}}{\Pi_{it-1}P_{jt-1}}\right)^{1-\sigma} + (1-\Omega_{jt-1})\left(\frac{\tau_{ijt-1}}{\Phi_{it-1}Z_{jt-1}}\right)^{1-\sigma}\right]^{\frac{1}{\alpha}}}\right]^{\alpha} \qquad (53)$$

Then using (22), along with the home price inflation rate ($\pi_{jt} = p_{jt}/p_{jt-1} - 1$), the capital growth rate ($\kappa_{jt} = K_{jt}/K_{jt-1} - 1$), and the market share ($s_{it} = Y_{it}/Y_t$), the above becomes:

$$Y_{jt} = \frac{(1+\pi_{jt})(1+\eta_{jt})^{1-\alpha}(1+\kappa_{jt})^{\alpha}\left(\frac{X_{ijt-1}}{s_{it-1}}\right)}{(\Omega_{jt-1})\left(\frac{\tau_{ijt-1}}{\Pi_{it-1}P_{jt-1}}\right)^{1-\sigma} + (1-\Omega_{jt-1})\left(\frac{\tau_{ijt-1}}{\Phi_{it-1}Z_{jt-1}}\right)^{1-\sigma}} \qquad (54)$$

Plugging (54) into (52) gives the *structural dynamic gravity equation* for $p_{it} \neq z_{it}$:

$$X_{ijt} = \frac{\left[X_{ijt-1}(1+\pi_{jt})(1+\eta_{jt})^{1-\alpha}(1+\kappa_{jt})^{\alpha}\left(\frac{s_{it}}{s_{it-1}}\right)\right]\left[\Lambda_{ijt}\right]}{(\Omega_{jt-1})\left(\frac{\tau_{ijt-1}}{\Pi_{it-1}P_{jt-1}}\right)^{1-\sigma}+(1-\Omega_{jt-1})\left(\frac{\tau_{ijt-1}}{\Phi_{it-1}Z_{jt-1}}\right)^{1-\sigma}} \quad (55)$$

Note that expressions (54) and (55) are generalizations and extensions that allow for $p_{it} \neq z_{it}$, which collapse to (30) and (32) when $p_{it} = z_{it}$.

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