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Sequential Theorems In Banach And 2- Banach Spaces For Fixed And Common Fixed Point Theorems

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ABSTRACT

In this present review article we have explored back to back theorems in Banach and 2- Banach space for unique fixed point and common fixed point with a contractive type condition which was the enhancement of well known results as well as we have applied two spaces for obtaining our result in first section the platform is Banach space and in other part our platform is 2- Banach space for single and multivalued mappings respectively.

KEY WORDS: Banach , 2- Banach spaces, Common fixed point theorem, , Expansion mappings , Fixed point theorem , Identity mapping .

1. INTRODUCTION

The notion of 2-Banach space was initiated by Gahler [7,8] in the year 1965 and enhanced by Iseki [9,10] with obtaining some results on fixed point theorems in 2-Banach spaces. After it many scientists , researchers and mathematicians have improvised , enhanced and demonstrated many fruitful results using various type of inequalities. By referring Brouwder [3] result it carried on by Sayyed et al [20,21], Sayyed and Badshah [19] and Jain and Sayyed [11] with various type of contractive conditions and found similar results which was used in this article . Continuing the same sequence in 2- Banach space authors namely white [26], Ahmed and Shakil [1], Khan & Imdad [13], Qureshi and Singh [15], Badshah & Gupta [2], Choudhary & Malviya [4], Som [23], Jong [12] and Datson [5] . A short time ago a full groom in 2- Banach space by Yadav et al [27], Dwavedi et.al.[6], Utpalendu and Hora Krishna [24] Saluja and Dhakde [16], Saluja [17], Malceski & Anevska [14], Vijayvargiya and Bharti [25], Shrivas [22] and Sarkar et.al.[18] with more significant and fertile result for development of advance Mathematics .

2. PRELIMINARY

In this article we shall use the following definitions which was defined by Gahler [7,8] .

DEFINITION 2.1: In mathematical analysis a Banach space is a complete normed vector space. That is, the distance between vectors converges closer to each other as the sequence goes on.

DEFINITION 2.2: Let X be a linear space and $\|.,.\|$ is a real valued function defined on X
where

(i) $\|a, b\| = 0$ if and only if a and b are linearly dependent ,

(ii) $\| a, b \| = \| b, a \|$,

(iii) $\| a, xb \| = |x| \| a, b \|$,

(iv) $\| a, b+c \| \leq \| a ,b \| + \| a,c \|$

For all a ,b, c ϵ X and x ϵ R. Then $\| .,. \|$ is called a 2-norm and the pair (X , $\| .,. \|$) is called a 2-norm space.

REMARK 2.1: In whole review article we denote X as a 2-normed space unless otherwise stated.

DEFINITION 2..3 : A sequence $\{x_n\}$ in a 2-norm space X is said to be convergent if there is a point x ϵ X such that $\lim_{n\to\infty} \| x_n - x, a \| = 0$ for all a ϵ X.

DEFINITION 2.4: A sequence $\{x_n\}$ in a 2-norm space X is called a Cauchy sequence if

$$\lim_{n,m\to\infty} \| x_n - x_m, a \| = 0 \text{ for all } a \in X.$$

DEFINITION 2.5: A linear 2-norm space is said to be complete if every Cauchy sequence in X is convergent in X. Then we say X is a 2-Banach Space.

3. MAIN RESULTS

THEOREM 3.1 : Let X be a Banach space and C be a closed and convex subset of X ,H be a mapping of a Banach space X and itself, if H satisfies the following conditions :

(C-1) $U^2 = I$ (Identity mapping)

(C-2) $$\|Hx - Hy\| \geq q\,[\|x - Hx\|\,\|y - Hy\| + \|y - Hx\|\,\|x - Hy\|] / \|x - y\| + q'\,[\|x - Hx\|\,\|x - Hy\| + \|y - Hy\|\,\|y - Hx\|] / \|x - y\| + r\,[\|x - Hx\|\,\|y - Hy\|] / \|x - y\| + r'\|x - y\|$$

Where x,y ϵ C , $x \neq y$ and q , q' , r and r' are non negative with $0 \leq 5q + 4q' + 4r + r' < 1$, then H has a unique fixed point .

PROOF : Suppose x is any point in Banach space and taking $y = \frac{1}{2}(H + I)x$ and z = Hy and u = 2y - z with using condition (C-2) ,we get

$$\begin{aligned}\|z - x\| &= \|Hy - H^2x\| \\ &= \|Hy - H(Hx)\| \\ &\geq q\,[\|y - Hy\|\,\|Hx - H(Hx)\| + \|Hx - Hy\|\,\|y - H(Hx)\|] / \|x - y\| \\ &\quad + q'\,[\|y - Hy\|\,\|y - H(Hx)\| + \|Hx - H(Hx)\|\,\|Hx - Hy\|] / \|x - y\| \\ &\quad + r\,[\|Hx - H(Hx)\|\,\|y - Hy\|] / \|x - y\| \\ &\quad + r'\|y - Hx\|\end{aligned}$$

By using (C-1) and assumed conditions , we write

$$\begin{aligned}&\geq q\,[\|y - Hy\|\,\|Hx - x\| + \tfrac{1}{4}\|Hx - x\|^2] / \tfrac{1}{2}\|Hx - x\| \\ &\quad + q'\,[\|y - Hy\|\,\tfrac{1}{2}\|Hx - x\| + \|Hx - x\|\,\tfrac{1}{2}\|Hx - x\|] / \tfrac{1}{2}\|Hx - x\| \\ &\quad + r\,[\|Hx - x\|\,\|y - Hy\|] / \tfrac{1}{2}\|Hx - x\| \\ &\quad + r'\tfrac{1}{2}\|x - Hx\|\end{aligned}$$

Or

$$\begin{aligned}\|z - x\| &\geq q\,[2\|y - Hy\| + \tfrac{1}{2}\|Hx - x\|] \\ &\quad + q'\,[\|y - Hy\| + \|Hx - x\|] \\ &\quad + r\,2\|y - Hy\| + r'\tfrac{1}{2}\|x - Hx\|\end{aligned}$$

Or

$$\|z - x\| \geq (2q + q' + 2r)\|y - Hy\| + (\tfrac{1}{2}q + q' + \tfrac{1}{2}r')\|Hx - x\| \qquad \text{\_ \_ \_ } (*)$$

Now for ,

$$\|u - x\| = \|2y - z - x\| = \|Hx - Hy\|$$

Using condition (C-2) , we get

$$\begin{aligned}&\geq q\,[\|x - Hx\|\,\|y - Hy\| + \|y - Hx\|\,\|x - Hy\|] / \|x - y\| \\ &+ q'\,[\|x - Hx\|\,\|x - Hy\| + \|y - Hy\|\,\|y - Hx\|] / \|x - y\| \\ &+ r\,[\|x - Hx\|\,\|y - Hy\|] / \|x - y\| + r'\|x - y\|\end{aligned}$$

$$\begin{aligned}&\geq q\,[\|x - Hx\|\,\|y - Hy\| + \tfrac{1}{2}\|x - Hx\|\,\tfrac{1}{2}\|x - Hx\|] / \tfrac{1}{2}\|x - Hx\| \\ &\quad + q'\,[\|x - Hx\|\,\tfrac{1}{2}\|x - Hx\| + \|y - Hy\|\,\tfrac{1}{2}\|x - Hx\|] / \tfrac{1}{2}\|Hx - x\| \\ &+ r\,[\|x - Hx, a\|\,\|y - Hy, a\|] / \tfrac{1}{2}\|Hx - x| + r'\tfrac{1}{2}\|Hx - x\|\end{aligned}$$

Or

$$\begin{aligned}&\geq q\,[2\|y - Hy\| + \tfrac{1}{2}\|x - Hx\|] \\ &\quad + q'\,[\|x - Hx\| + \|y - Hy\|] + r\,2\|y - Hy\| \\ &\quad + r'\tfrac{1}{2}\|Hx - x\|\end{aligned}$$

Or

$$\|u - x\| \geq (2q + q' + 2r)\|y - Hy\| + (\tfrac{1}{2}q + q' + \tfrac{r'}{2})\|x - Hx\| \qquad \text{\_ \_ \_ } (**)$$

Now

$\|z - u\| = \|z - x\| + \| x - u \|$, then by equations (*) and (**) , we have

$$\|z - u| \geq (4q + 2q' + 4r)\|y - Hy\| + (q + 2q' + r')\|x - Hx\|$$

$$\_\_\_ (***)$$

On other hands

$\|z - u\| = \|Hy - (2y - z)\| = \| Hy - 2y + Hy \| = 2\|Hy - y\|$

by equation (***), we get

$2\|Hy - y\| \geq (8q+4q'+8r)\|y - Hy\| + (2q+4q'+2r' \|x - Hx\|$

$(2 - 8q - 4q' - 8r)\|y - Hy, a\| \geq (2q + 4q' + 2r') \|x - Hx\|$

Or $\|x - Hx, a\| \leq \frac{2-8q-4q'-8r}{2q+4q'+2r'} \|y - Hy\|$

Or $\|x - Hx\| \leq k \|y - Hy\|$, where $k = \frac{2-8q-4q'-8r}{2q+4q'+2r'} < 1$

Let $R = \frac{1}{2}(H + I)$, then

$\|R^2x - Rx\| = \| RR(x) - Rx \|$

$= \|Ry - y\| = \frac{1}{2} \|y - Hy\| < \frac{k}{2} \|x - Hx\|$

by the definition of R we claim that $\{R^n x\}$ is a Cauchy sequence in X . $\{R^n x\}$ is converges to a element x_0 in X , so $\lim_{n\to\infty} \{R^n x\} = x_0$, so $\{Rx_0\} = x_0$.
Hence $Hx_0 = x_0$.

UNIQUENESS : If possible $y_0 \neq x_0$ is a another fixed point of H, then

$\|x_0 - y_0, a\| = \|Hx_0 - Hy_0, a\|$, then by using condition (C-2) , we have

$\geq q[\|x_0 - Hx_0\| \| y_0 - Hy_0\|$

$+\| y_0 - H x_0 \| \| x_0 - Hy_0\|] / \| x_0 - y_0 \|$

$+ q' [\|x_0 - Hx_0\| \| x_0 - Hy_0\|$

$+ \| y_0 - Hy_0 \| \| y_0 - Hx_0\|] / \| x_0 - y_0\|$

$+ r [\|x_0 - Hx_0\| \| y_0 - Hy_0\|] + r' \| x_0 - y_0\|$

Or $\| x_0 - y_0 \| \geq (q + r') \| x_0 - y_0 \|$, which is a contradiction , hence $y_0 = x_0$. It is clear that fixed point is unique.

THEOREM 3.2 : Let H be a mapping of a 2- Banach space X into itself, if H satisfies the following conditions :

(C-1.1) $H^2 = I$ (Identity mapping)

(C-1.2) $\|Hx - Hy, a\| \geq q [\|x - Hx, a\| \|y - Hy, a\|$

$+ \|y - Hx, a\| \|x - Hy, a\|] / \|x - y, a\|$

$+q [\|x - Hx, a\| \|x - Hy, a|$

$+ \|y - Hy, a\| \|y - Hx, a\|] / \|x - y, a\|$

$+ r [\|x - Hx, a\| \|y - Hy, a\|] + r' \|x - y, a\|$

Where $x \neq y$, $x.y \in X$ and q , q' , r and r' are non negative with $0 \leq 5q + 4q' + 4r + r' < 1$, then H has a unique fixed point .

PROOF : Same as assuming by previous theorem without change and same the proof . Let a_1 and a_2 be two linearly independent vectors of X. Then by proof of pervious theorem we can write

$$\|R^2x - Rx, a_1\| \leq \frac{k}{2} \|x - Hx, a_1\| , \quad - - - (1)$$

And $$\|R^2x - Rx, a_2\| \leq \frac{k}{2} \|x - Hx, a_2\| , \quad - - - (2)$$

Inequalities (1) and (2) together imply that the sequence $\{R^n x\}$ is a Cauchy sequence in X and X is complete then $\{R^n x\}$ is converges to a element x_0 in X , so $\lim_{n\to\infty} \{R^n x\} = x_0$, so $\{Rx_0\} = x_0$. Hence $Hx_0 = x_0$.

Similarly as by previous theorem proof , we claim that for uniqueness that

$\| x_0 - y_0, a\| \geq (q + r') \| x_0 - y_0, a\|$, which is a contradiction , hence $y_0 = x_0$. It is clear that fixed point is unique.

This completes the proof .

THEOREM 3.3 : Let X be a Banach space and C be a closed and convex subset of X ,H and I be mappings of a Banach space X and itself, if H and I satisfy the following conditions :

(C-1) H and I are commute ,

(C-2) $H^2 = I$ and $I^2 = I$, where I is identity mapping

(C-3) $\| Hx - Hy \| \geq q [\| Ix - Hx \| \|Iy - Hy\|$

$+ \|Iy - Hx\| \|Ix - Hy\|] / \|Ix - Iy\|$

$+ q' [\|Ix - Hx\| \|Ix - Hy\|$

$+ \|Iy - Hy\| \|Iy - Hx\|] / \|Ix - Iy\|$

$+ r [\|Ix - Hx\| \|Iy - Hy\|] + r' \|Ix - Iy\|$

for all x,y ϵ X , $x \neq y$ and p,q,r and r' are non negative with $0 \leq 5q + 4q' + 4r + r' < 1$

and $\| Ix - Iy \| \neq 0$ then there exists a unique common fixed point of H and I such that $H(x_0) = x_0$ and $I(x_0) = x_0$.

PROOF : Suppose x is a point in Banach space then clear that $(HI)^2 = I$. Now by using condition (C-3) , we have

$$\| HI(Ix) - HI(Iy) \| \geq q [\| I(I^2x) - H(I^2x) \| \, \|I(I^2y) - H(I^2y)\|$$

$$+ \|I(I^2y) - H(I^2x) \| \, \|I(I^2x) -H(I^2y) \|] / \|I(I^2x) - I(I^2y)\|$$

$$+ q'[\|I(I^2x) - H(I^2x) \| \, \|I(I^2x) - H(I^2y)\|$$

$$+ \|I(I^2y) - H(I^2y) \| \, \|I(I^2y) - H(I^2x)\|] / \|I(I^2x) -I(I^2y) \|$$

$$+ r[\|I(I^2x) - H(I^2x)\|\|I(I^2y) - H(I^2y) \|]+ r' \|I(I^2x) -I(I^2y) \|$$

$$\| HI(Ix) - HI(Iy) \| \geq q [\| Ix - HI(Ix) \| \, \|Iy - HI(Iy)\| + \|Iy - HI(Ix) \| \, \|Ix -HI(Iy) \|] / \|Ix - Iy\|$$

$$+ q' [\|Ix - HI(Ix)\| \, \|Ix - HI(Iy) \|$$

$$+ \|Iy - HI(Iy) \| \, \|Iy - HI(Ix)\|] / \|Ix -Iy \|$$

$$+ r [\|Ix - HI(Ix) \| \, \|Iy - HI(Iy) \|] + r' \|Ix -Iy \|$$

Taking Ix = e and Iy = f , then

$$\| HI(e) - HI(f) , a\| \geq q [\| e - HI(e) \| \, \|f - HI(f) \|$$

$$+ \|f - HI(e)\| \, \|e -HI(f) \|] / \|e - f \|$$

$$+ q'[\|e - HI(e) \| \, \|e - HI(f) \|$$

$$+\|f - HI(f) \| \, \|f - HI(e)\|] / \| e -f \|$$

$$+ r [\|e - HI(e)\| \, \| f - HI(f) \|] + r' \|e -f \|$$

It is clear by previous theorem that R = HI has at least one fixed point say x_0 in K that is

$$Rx_0 = HIx_0 = x_0 ,$$

and $$H(HI)x_0 = Hx_0$$

Or $$H^2 = (Ix_0) = Hx_0$$

$$Ix_0 = Hx_0$$

now

$$\|Hx_0 - x_0 \| = \| Hx_0 - H^2x_0 \| = \| Hx_0 - H(Hx_0) \|$$

$$\geq q [\|x_0 - Hx_0 \| \, \|Hx_0 - HHx_0 \|$$

$$+ \|Hx_0 - Hx_0 \| \, \|x_0 -HHx_0 \|] / \|x_0 - Hx_0 \|$$

$$+ q' [\|x_0 - Hx_0 \| \, \|x_0 - HHx_0 \|$$

$$+ \|Hx_0 - HHx_0 \| \, \|Hx_0 - Hx_0 \|] / \|x_0 -Hx_0 \|$$

$$+ r [\|x_0 - Hx_0 \| + \|Hx_0 - HHx_0 \|] + r' \|x_0 - Hx_0\|$$

$$\|Hx_0 - x_0 ,a\| \geq (q' + r') [\|x_0 - Hx_0 ,a\|$$

Or $x_0 = Hx_0$. Hence x_0 is a fixed point in H, but $Hx_0 = Ix_0$ so, $Ix_0 = x_0$

Hence x_0 is a common fixed point of H and I.

UNIQUENESS : If possible $y_0 \neq x_0$ is a another fixed point of H and I, then

$\|x_0 - y_0 \| = \|H^2x_0 - H^2y_0 \| = \|H(Hx_0) - H(Hy_0)\|$ then by using condition (C-3) , we have

$$\geq q [\| I(Hx_0) - H(Hx_0)\| \, \|I(Hy_0) - H(Hy_0) \|$$
$$+\|I(Hy_0) - H(Hx_0)\| \, \|I(Hx_0)-H(Hy_0)\|] / \|I(Hx_0) - I(Hy_0)\|$$
$$+ q' [\|I(Hx_0) - H(Hx_0) \| \, \|I(Hx_0) - H(Hy_0) \|$$
$$+ \|I(Hy_0) - H(Hy_0) \| \, \|I(Hy_0) - H(Hx_0)\|] / \|I(Hx_0) - I(Hy_0) \|$$
$$+ r [\|I(Hx_0) - H(Hx_0) \| \, \|I(Hy_0) - H(Hy_0) \|]$$
$$+ r' \|I(Hx_0) -I(Hy_0) \|$$

Or

$$\|x_0 - y_0 \| \geq (q' + r') \|x_0 - y_0\|$$

Hence $x_0 = y_0$, common fixed point is unique.

Or $\| x_0 - y_0 \| \geq (q' + r') \| x_0 - y_0 \|$, which is a contradiction , hence $y_0 = x_0$. It is clear that fixed point is unique.

THEOREM 3.4 : Let H and I be two expansion mappings of a 2- Banach space X into itself and H and I satisfying the following conditions ,

(C-1) H and I are commute ,

(C-2) $H^2 = I$ and $I^2 = I$, where *I* is identity mapping

(C-3) $$\| Hx - Hy , a\| \geq q [\| Ix - Hx ,a\| \, \|Iy - Hy , a\| + Hx , a\| \, \|Ix -Hy , a\|] / \|Ix - Iy,a\|$$

$$+ q' [\|Ix - Hx ,a\| \, \|Ix - Hy , a\|$$

$$+ \|Iy - Hy , a\| \, \|Iy - Hx , a\|] / \|Ix -Iy , a\|$$

$$+ r [\|Ix - Hx ,a\| + \|Iy - Hy , a\|] + r' \|Ix -Iy ,a\|$$

Where $x \neq y$, $x.y \in X$ and q , q' , r and r' are non negative with $0 \leq 5q + 4q' + 4r + r' < 1$, then H has a unique fixed point and $\| Ix - Iy \| \neq 0$.

PROOF : Same as assuming by previous theorem without change and same the proof . Let a_1 and a_2 be two linearly independent vectors of X. Then by proof of pervious theorem we can write

$$\|R^2x - Rx , a_1 \| \leq \frac{k}{2} \|x - Hx , a_1 \| , \quad (1)$$

And $$\|R^2x - Rx , a_2 \| \leq \frac{k}{2} \|x - Hx , a_2 \| , \quad (2)$$

Inequalities (1) and (2) together imply that the sequence $\{R^nx\}$ is a Cauchy sequence in X and X is complete then $\{R^nx\}$ is converges to a element x_0 in X , so $\lim_{n\to\infty} \{R^nx\} = x_0$, so $\{Rx_0\} = x_0$. Hence $Hx_0 = x_0$

Similarly as by previous theorem proof , we claim that for uniqueness that

$\| x_0 - y_0 , a \| \geq (q' + r') \| x_0 - y_0 , a \|$, which is a contradiction , hence $y_0 = x_0$. It is clear that fixed point is unique.

This completes the proof .

CONCLUSION

In this paper, proved a unique fixed point theorem as well as common fixed point theorem by using contractive type inequality in Banach and 2-Banach space . These results can be extended to any directions and can also be extended to fixed point theory of single-valued and multivalued mappings.

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