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I-Scheme With Contractive Condition In Hilbert Space And Common Fixed Point Theorem

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ABSTRACT: In our ongoing review write-up, entrenched a common fixed point theorem for generalized contraction with I-scheme in Hilbert space by using rational expression type condition. Our evaluation is conception of prior set out come with rectifying existence and uniqueness of fixed point theory . We have taken advantages of preceding known authors done work as well as Ishikawa scheme for getting more significant result.

KEYWORDS: Closed Convex Subset , Fixed Point Theory, Hilbert Space, I-Scheme, Rational Expression.

1. INTRODUCTION AND PRELIMINA

The supposition of generalized contraction mapping was put forwarded by Ciric [1] and elaborated many ideas for it. Further Das and Gupta [2] implemented it in reflexive Banach space. Naimpally and Singh [6] extended the results of Rhoades [11,12] then Sayyed and Badshah [14] forwarded the result of Naimpally and Singh by using contraction condition.

Definition 1.1. A vector space furnished with metric space as well as inner product space with associated norm is known as Hilbert space that is every Cauchy sequence in Hilbert space has a limit in Hilbert space

Definition 1.2 . A Hilbert space is a complete inner product space.

Note1.1. In a Hilbert space limit of a Cauchy sequence is unique.
Note 1.2. Hilbert space is a vector space .

We pursuance the results and the iteration scheme called I- Scheme of Sayyed and Badshah [13,14,15,16] and authors namely Imdad and Ali [3] , Yadav et,al [18], Veerapandi and Kumar [17],Rao et.al.[9] , Rao and Kalyani [10], Nigam et.al. [7], Park [8]. Recently Jain and Sayyed [4] and Mane and

Sayyed [5] shown fixed point theory in contractive type condition

2. RESULTS

THEOREM 2.1 : Let H be a Hilbert space and let C be a closed convex subset or H. Let R and S be two mappings satisfying

$$||Rx-Sy||^2 \le a\,||x-y||^2 + a^* \frac{||y-Sy||^2[1+||x-Rx||^2]}{1+||x-y||^2}$$

$$+ b\,\frac{[1+||y-Sy||^2]\,||x-Rx||^2}{1+||x-y||^2}$$

$$+ c\,[||x-Rx||^2 + ||y-Sy||^2]$$

$$+ c^*\,[||x-Sy||^2 + ||y-Rx||^2] \quad \text{.. .... (2.1)}$$

Here a,a*,b,c and c*are non zero with $0 \le 4a+4a^*+4b+8c+6c^* \le 1$. If ∃ a point x_0 such that the I- scheme for R and S defined by as same Sayyed and Badshah [15] converges to a point u, then u is a common point of R and S.

PROOF:

By Sayyed and Badshah [14] that

$x_{2n+1} - x_{2n} = \alpha_{2n}(Sy_{2n} - x_{2n})$

Since $x_{2n} \to u$, $\| x_{2n+1} - x_{2n} \| \to \infty$

Since { α_{2n} } is bounded away from zero,

$||Sy_{2n} - x_{2n}|| \to 0$ as $n \to \infty$.

It follows that $||u - Sy_{2n}|| \to 0$ as $n \to \infty$.

Equation (2.1) satisfied by R and S, then

$$||Rx_{2n}-Sy_{2n}||^2 \le a\,||x_{2n} - y_{2n}||^2$$

$$+ a^* \frac{||y_{2n}-Sy_{2n}||^2[1+||x_{2n}-Rx_{2n}||^2]}{1+||x_{2n}-y_{2n}||^2}$$

$$+ b\,\frac{[1+||y_{2n}-Sy_{2n}||^2]\,||x_{2n}-Rx_{2n}||^2}{1+||x_{2n}-y_{2n}||^2}$$

$$+ c\,[||x_{2n}-Rx_{2n}||^2 + ||y_{2n}-Sy_{2n}||^2]$$
$$+ c^*\,[||x_{2n}-Sy_{2n}||^2 + ||y_{2n} - Rx_{2n}||^2]$$

.....(2.2)

Now, $||y_{2n} - x_{2n}||^2 = \| \beta_{2n} Rx_{2n} + (1-\beta_{2n})\,x_{2n} - x_{2n} \|^2$

$$= \| \beta_{2n} Rx_{2n} + x_{2n} - \beta_{2n}x_{2n} - c \|^2$$

$$= \| \beta_{2n}(Rx_{2n} - x_{2n}) \|^2$$

$$= \beta^2_{2n} \| (Rx_{2n} + Sy_{2n}) + (Sy_{2n} - x_{2n}) \|^2$$

$$\le 2\,||Rx_{2n} - Sy_{2n}||^2 + 2\,\|Sy_{2n} - x_{2n}||^2$$

..........(2.3)

and

$$||y_{2n} - Sy_{2n}||^2 = \| \beta_{2n} Rx_{2n} + (1-\beta_{2n})\,x_{2n} - Sy_{2n}||^2$$

$$= \| \beta_{2n} Rx_{2n} + (1-\beta_{2n})\,x_{2n} - Sy_{2n} + \beta_{2n} Sy_{2n} - \beta y_{2n} Sy_{2n} ||^2$$

$$= \| \beta_{2n}(Rx_{2n} - Sy_{2n}) + (1-\beta_{2n})(x_{2n} - Sy_{2n})||^2$$

$$\le 2\,\beta^2_{2n} \| Rx_{2n} - Sy_{2n}||^2 + 2\,(1-\beta_{2n})^2 \| x_{2n} - Sy_{2n}||^2$$

$$\le 2\,\| Rx_{2n} - Sy_{2n}||^2 + 2\,\| x_{2n} - Sy_{2n}||^2$$

.......(2.4)

from (2.2), (2.3) , (2.4) can be written as:

$$||Rx_{2n}-Sy_{2n}||^2 \le a\,[2\,||Rx_{2n} - Sy_{2n}||^2 + 2\,\| Sy_{2n} - x_{2n}||^2]$$

$$+ a^* \frac{[2||Rx_{2n}-Sy_{2n}||^2 + 2\,||x_{2n}-Sy_{2n}||^2][1+2||x_{2n}-Sy_{2n}||^2+2||Sy_{2n}-Rx_{2n}||^2]}{1+2\,||Rx_{2n}-Sy_{2n}||^2 + 2||Sy_{2n}-x_{2n}||^2}$$

$$+ b\,\frac{\left[1+2||Rx_{2n}-Sy_{2n}||^2 + 2||x_{2n}-Sy_{2n}||^2\right][2\,||x_{2n}-Sy_{2n}||^2+2||Sy_{2n}-Rx_{2n}||^2)}{1+2\,||Rx_{2n}-Sy_{2n}||^2 + 2||Sy_{2n}-x_{2n}||^2}$$

$$+ c\,[2||x_{2n}-Sy_{2n}||^2 + 2\,||Sy_{2n}-Rx_{2n}||^2 + 2\,\| Rx_{2n} - Sy_{2n}||^2 + 2\,\| x_{2n} - Sy_{2n}||^2]$$
$$+ c^*\,[||x_{2n}-Sy_{2n}||^2 + 2\,\| Rx_{2n} - Sy_{2n}||^2 + 2\,\| x_{2n} Sy_{2n}||^2]$$

Or

$$||Rx_{2n}-Sy_{2n}||^2 \le a\,[2\,||Rx_{2n} - Sy_{2n}||^2 + 2\,\| Sy_{2n} - x_{2n}||^2]$$

$$+a^*\,[2||Rx_{2n} - Sy_{2n}||^2 + 2\,||x_{2n} - Sy_{2n}||^2]$$

$$+ b\,[2||x_{2n}-Sy_{2n}||^2 + 2\,||Sy_{2n}-Rx_{2n}||^2]$$
$$+c\,[4||x_{2n}-Sy_{2n}||^2 + 4||Rx_{2n}-Sy_{2n}||^2]$$

$$+ c^*\,[3||x_{2n}-Sy_{2n}||^2 + 2\,\| Rx_{2n} - Sy_{2n}||^2]$$

Or

$||Rx_{2n}-Sy_{2n}||^2 \leq (2a + 2a^*+2b+4c+3c^*) ||Rx_{2n}-Sy_{2n}||^2$

$+ (2a+2a^*+2b+4c+3c^*) ||x_{2n}-Sy_{2n}||^2$

Or

$(1-2a-2a^*-2b-4c-3c^*) \; ||Rx_{2n}-Sy_{2n}||^2 \leq$

$(2a+ 2a^*+2b+4c + 3c^*) ||x_{2n}-Sy_{2n}||^2$

Or

$||Rx_{2n}-Sy_{2n}||^2 \leq \frac{2a+2a^*+2b+4c+3c^*}{1-2a-2a^*-2b-4c-3c^*} \; ||x_{2n}-Sy_{2n}||^2$

Taking the lim as n→ ∞, we get $||Rx_{2n} - Sy_{2n}||^2 \to 0$. It follows that

$||x_{2n} - Rx_{2n} \; ||^2 \leq ||x_{2n} - Sy_{2n} \; ||^2$

$+ 2||Sy_{2n} - Rx_{2n} \; ||^2 \to 0$

and,

$||u- Rx_{2n}||^2 \leq 2||u - x_{2n}||^2 + 2 \; ||x_{2n} - Sy_{2n} \; ||^2 \to 0 \; as \; \to \infty$.

If x_{2n} and u satisfy (2.1) we have

$||Rx_{2n}-Su||^2 \leq a||x_{2n} -u \; ||^2$

$+ a^* ||u-Su||^2 [1+ ||x_{2n}-Rx_{2n}||^2] \; / \; 1+ [||x_{2n}-u||^2$

$+ b[1+ ||u-Su||^2] ||x_{2n}-Rx_{2n}||^2 \; / \; 1+ [||x_{2n}-u||^2$

$+c \; [||x_{2n}-Rx_{2n}||^2 + ||u-Su||^2]$

$+ c^* \; [||x_{2n}-Su||^2 + ||u - Rx_{2n}||^2]$

$||Rx_{2n}-Su||^2 \leq a||x_{2n} -u \; ||^2 +$

$a^* \; [2|| u- Rx_{2n}||^2 +2|| Rx_{2n} - Su||^2] \; [1+ ||x_{2n}-Rx_{2n}||^2] \; / \; 1+ [||x_{2n}-u||^2$

$+ b[\; 1+ 2|| u- Rx_{2n}||^2 +2|| Rx_{2n} - Su||^2 \;] \; ||x_{2n}-Rx_{2n}||^2 \; / \; 1+ [||x_{2n}-u||^2$

$+c \; [||x_{2n}-Rx_{2n}||^2 + 2|| u- Rx_{2n}||^2 +2|| Rx_{2n} - Su||^2 \;] + c^* \; [||x_{2n}-Su||^2 + ||u - Rx_{2n}||^2]$

Taking the lim as n→ ∞, we obtain

$||Rx_{2n} - Su||^2 \leq (2a^*+2c) \; ||Rx_{2n} - Su||^2$

$(1-2a^*-2c) \; ||Rx_{2n} - Su||^2 \leq 0.$

that is, $||Rx_{2n} - Su||^2 \to 0$

Finally $||u-Su||^2 = || \; u - Rx_{2n} +Rx_{2n} - Su||^2$

$\leq 2|| \; u - Rx_{2n} \; ||^2 + || \; Rx_{2n} - Su||^2 \to 0$ as n→ ∞,

It shows that u= Su.

Proceeding in the same manner ; we can write

u= Ru.

Thus, u is a common fixed point of R and S. This complete the proof of theorem.

Letting R = S = Z in above theorem, we obtain the following corollary :

COROLLARY 1.1 : Let B be a Hilbert space , C be a closed convex subset of Hilbert space H and Z be a self mapping in Hilbert space into itself satisfying condition 2.1 and a,a*,b,c and c*are non zero with

$0 \leq 4a+4a^*+4b+8c+6c^* \leq 1.$

where $0 \leq 4a+4a'+4b+8c+6c' \leq 1.$

If there exists a point x_0 such that the I-scheme for Z defined by

$y_n = \beta_n \, Z_{xn} + (1- \beta_n) \, x_n, \; n \geq 0$

$x_{n+1} = (1- \alpha_n) \, x_n + \alpha_n \, Z_{yn} \; , \; n \geq 0$

converges to a point p, then p is the fixed point of Z.

In the I- scheme ,{ α_n }, { β_n } satisfy $0 \leq \alpha_n \leq \beta_n \leq 1$ for all n.

$\lim_{n\to\infty} \beta_n. \sum \alpha_n\beta_n = 0$ Assuming that

(i) $0 \leq \alpha_n, \beta_n \leq 1$, for all n.

(ii) $\lim \alpha_n = \alpha > 0$,

(iii) $\lim \beta_n = \beta > 1$.

The proof is similar to above Theorem, Hence we omit the details.

3. CONCLUSION

In this work ,we have obtained a unique common fixed point theorem in Hilbert space with I-scheme using rational type inequality .Our done work is an expedition of many previous known results .

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