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Simplification 3d Points Cloud Method Based On Importance Of 3d Points

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ABSTRACT- Representing the surface of complex objects, the samples resulting from their digitization can contain a very large number of points, endorsing simplification techniques, which analyse the relevance of the data. Simplification techniques also provide models with fewer points than the original ones. On the contrary the reconstruction of a surface, with simplified point cloud, must be close to the original. In this article, we develop a method of simplification reduce the number of scanned dense points, based on the density estimation and the notion of entropy. The performance of this approach is illustrated through experimental results and comparison with other simplification methods.

Keywords: Simplification; 3D Point Cloud; Shannon's entropy, K-Nearest Neighbours Estimator

1. INTRODUCTION

Modern three-dimensional (3D) scanning devices such as 3D scanners are highly developed. This evolution is mainly at the level of the resolution and the quality of the 3D point clouds obtained from the digitization of the real objects. The point cloud obtained using these devices has a data redundancy that makes calculations more difficult during reconstruction phases. To remedy this problem, we can go through a phase of simplification of the point cloud. This simplification step makes it possible to reduce the number of 3D points of an original X point cloud to obtain a simplified point cloud X' with an optimal number of points, such that X' is close to X. In the literature, there are two main categories of simplification algorithms: subsampling methods [1] and resampling methods or a combination of them [2],[3].

Subsampling algorithms produce a simplified point sample that is a subset of the original point cloud while resampling algorithms rely on estimating properties of the sampled surface to compute new relevant points.

several simplification techniques and methods are presented such as the method of Pauly et al. [4] used hierarchical decomposition of a points set. Linsen [5] presented a technique based on the measurement of the average variation of information such as direction of normal. Dey et al. [6] proposed a method based on a local feature size of the set of points. Alexa et al. [7] presented a method based on moving least squares to estimate the local geometrical properties of the sampled surface. Xuan W. et al [8] presented a method that used information theory and normal angle.

In subsampling method, there are three simplification schemes: the first method is selection of points representing subsets. This simplification strategy consists in breaking down the sample of points into small regions then each one of these regions is presented with a point [4],[9].

The second method is iterative simplification. This category involves deleting the points incrementally. This strategy is based on criteria that locally measure data redundancy [6],[7][10][11].

The third method is simplification by incremental sampling. For this approach, the simplified sample is constructed by incrementally enriching an initial subset of points or sampling an implicit surface [12]-[13].

In this paper, we inspired by the work of Jing Zang et al. [14] and we suggest an iterative simplification technique to choose the representative data points and remove redundant points. This method is based on the density estimation and the notion of entropy.

This paper is organized as the following. In section 2, we evoke the density function estimator and entropy definition. Then, in section 3, we present how to evaluate simplified meshes. Afterwards, in section 4, we lay out our 3D point cloud simplification algorithm based on the Shannon's entropy [15]. While section 5, lay out the experimental results and the validation. Finally, we wrap up with a conclusion.

2. ESTIMATION OF DENSITY FUNCTION AND ENTROPY DEFINITION

There are several methods of density estimation such as parametric and nonparametric methods. Parametric approach estimates the probability density function by assuming a parameterized model and then estimating the model parameters such as maximum likelihood estimator [16]. Nonparametric methods include the kernel density estimator, also known as the Parzen-Rosenblatt method [17],[18] and the K nearest neighbors (K-NN) method [19], each type has its advantages and disadvantages. In Parzen estimator, the bandwidth choice has strong impact on the quality of the estimated density [20],[21]. In this paper, we will use a K-NN estimator to estimate the density function.

2.1. The K Nearest Neighbors Estimator

The K nearest neighbors (K-NN) algorithm [19],[20] is a nonparametric probability density estimation. The control of the degree of the estimation is done by a scaler number k which is the number of the nearest neighbors. The coefficient k is generally proportional to the sample size N.

For each x where we estimate the density, we define distances between points of the sample and the x as following:

$$r_1(x) < \cdots < r_{k-1}(x) < r_k(x) < \cdots < r_n(x) \quad (1)$$

These distances are arranged in ascending order. The estimator with the method of nearest neighbour in dimension d can be defined as follows:

$$P_{knn}(x) = \frac{k/N}{V_k(x)} = \frac{k/N}{C_d r_k(x)} \quad (2)$$

where $r_k(x)$ is the distance from x to the kth nearest point, $V_k(x)$ is the volume of a sphere of radius $r_k(x)$ and C_d is the volume of the unit sphere in d dimension.

The adjustment of the number k must be a function of the size N of the available sample to respect the constraints that ensure the convergence of the estimator. For a number N of observations, the number k can be calculated as follows [22]:

$$k = k_0.\sqrt{N} \quad (3)$$

By respecting these rules of adjustment, it is certain that the estimator converges when the number k increases indefinitely, whatever the value of k_0.

2.2. Entropy definition

The concept of entropy associated with an X random variable is introduced by Claude Shannon as a basic concept of information theory [15]. There are two methods for measuring entropy.

2.2.1. Shannon's entropy

Let the distribution of probabilities $p = \{p_1, p_2, \ldots, p_N\}$ associated with the realizations of X. The Shannon entropy is calculated using the formula:

$$\mathbf{H(X)} = -\sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{p_i \log(p_i)} \quad (4)$$

2.2.2. Rényi's entropy

Rényi's entropy is a mathematical generalization of Shannon's entropy. Given a discrete random variable X with n possible values $(x_1, x_2, \ldots, x_n)$ as well as a strictly positive real parameter α and should be different from 1 [23][24].

The Rényi's entropy of order α of X is defined by the formula:

$$H_\alpha(X) = \frac{1}{1-\alpha} log \sum_{i=1}^{n} P(X = x_i)^\alpha \quad (5)$$

In rest of this paper, we use the Shannon's entropy.

3. ASSESSMENT OF THE SIMPLIFIED MESHES

A theoretical evaluation of the simplification method is based on a metric of mean errors, max error and RMS (Root Mean Square Error) used by Cignoni et al. [25]. In this context, Cignoni et al. have measured the Hausdorff distance [26] between the approximation and the original model. Hausdorff distance is defined as follow:

Let X and Y be two non-empty subsets of a metric space (M, d). Then, we define Hausdorff distance in the following way:

$$d_H = \max \begin{Bmatrix} \sup_{x\in X} \inf_{y \in Y} d(x,y), \\ \sup_{y\in Y} \inf_{x \in X} d(x,y) \end{Bmatrix} \quad (6)$$

where *Sup* represents the supremum and *inf* represents the infimum, and $d(.,.)$ indicates Euclidean distance in $\mathbb{R}^3$. In our experiments, we have used the symmetric Hausdorff distance calculated with the Metro software tool[25].

To calculate the approximate error, we will reconstruct the models from the point clouds as well as to create a 3D model from a set of points.

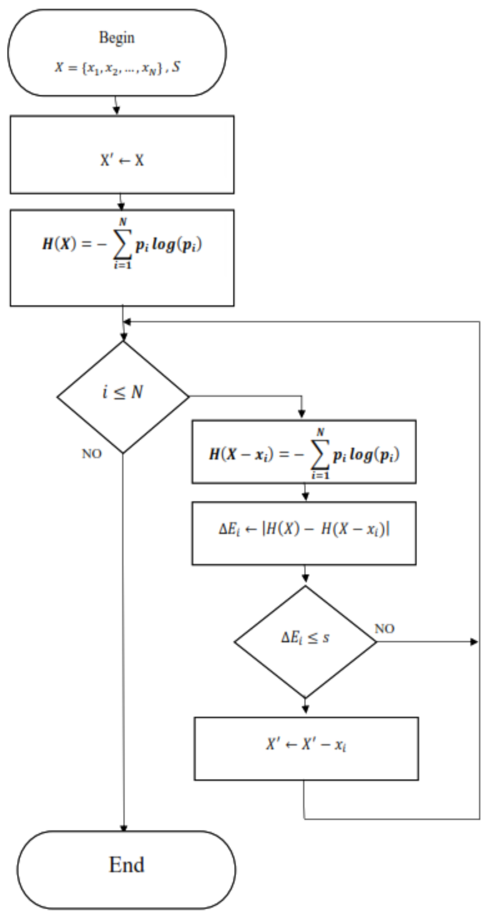
4. PROPOSED SIMPLIFICATION APPROACH

The goal of simplification method is to pick out representative data points and remove redundant points. In this paper, a simplification technique is developed to simplify data points. It is based on the K-NN density estimation and the Shannon's entropy.

We consider a 3D point cloud $X = \{x_1, x_2, \ldots, x_N\}$ at the input (see Figure 1). The suggested method begins with entropy estimation using all the data points of X, denoted by H(X). As previously indicated, the estimation of the density function is done by a K-NN estimator. Next, we estimate the entropy designated by $H(X\text{-}x_i)$ using $X - \{x_i\}$ with $i = 1, \ldots, N$. Then, we compute the difference between $H(X)$ and $H(X - x_i)$ denoted by $\Delta E_i = |H(X) - H(X - x_i)|, i = 1, \ldots, N$.

Afterwards, for $\Delta E_i \leq s$, s is the known threshold chosen by user, the point x_i will be removed from point cloud X. If not, the point x_i will be retained. At the end of the simplification algorithm, we obtain a simplified point cloud X' (see Figure 2).

The proposed simplification organizational charts is described as follows:



We note that the level of simplification of our approach is determined by the user. The level is presented by threshold s. In this paper to perform the calculations we use $s = 0.01$.

5. EXPERIMENTAL RESULTS AND COMPARISON

5.1. Experimental results

We illustrate our simplification approach using three 3D model that represent a synthetic 3D model such as a torus (figure 1,a). We Also use real objects such as, tennis shoe (figure 1,b), Hugo (figure 1,c) and Fandisck (figure 1,d). Figures (2,a), (2,b), (2,c) and (2,d) show the results of the simplification on various sample points.

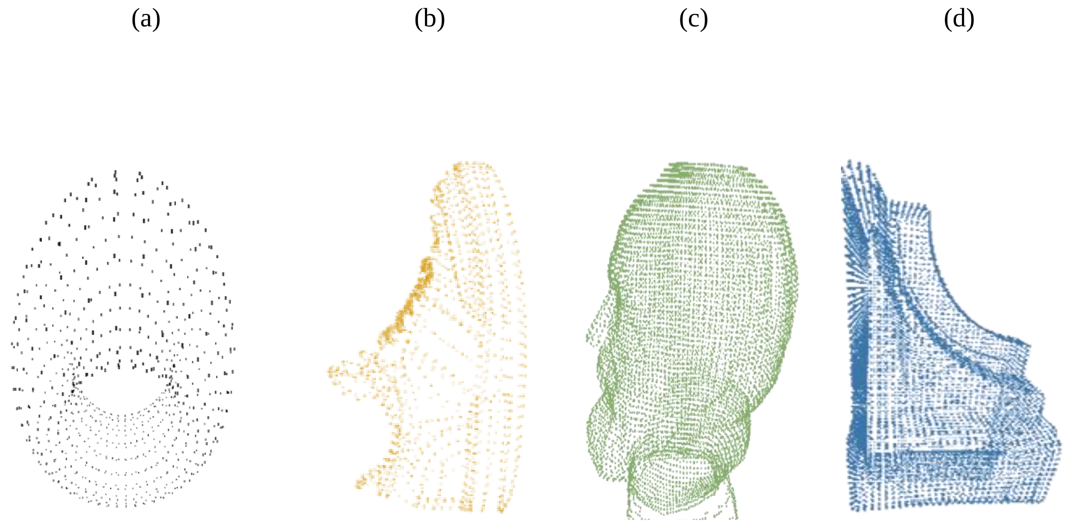


Figure 1: original point cloud: a) torus, b) tennis shoe, c) Hugo, d) Fandisck

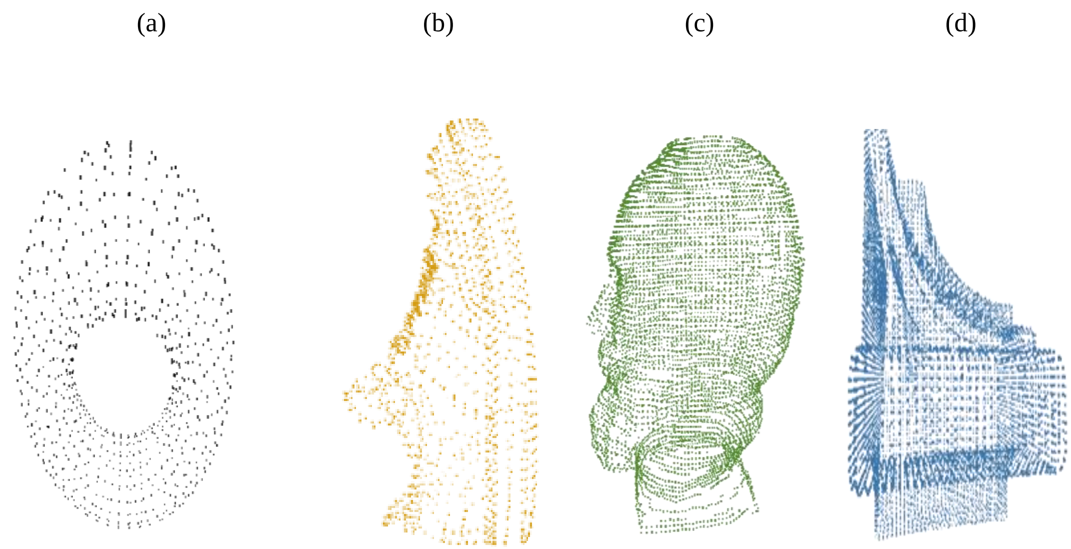


Figure 2: simplified point cloud: a) torus, b) tennis shoe, c) Hugo, d) Fandisck

In the next section, we validate the effectiveness of our proposed method through experimental results. In the first stage we conduct a comparison between the original and the simplified point cloud. In the next stage, we compare our method with the Allègre's method [11].

5.2. Comparison between original and simplified mesh

In this section, we make a comparison between the original mesh and the one created from the simplified point cloud. To reconstruct the mesh, we use ball Pivoting method [27],[28], method [29]. Then to measure the quality of the obtained meshes, we compute the quality of the triangles using the formula of compactness proposed by Gueziec et al. [30]:

$$c = \frac{4\sqrt{3}a}{l_1^2 + l_2^2 + l_3^2} \quad (7)$$

where l_i constitute the lengths of the edges of a triangle and a compose its area. We observe that this measure is equal to 1 for an equilateral triangle and 0 for a triangle whose vertices are collinear. According to [30], a triangle is of acceptable quality if $c \geq 0.6$. The following figure shows how to calculate the compactness.

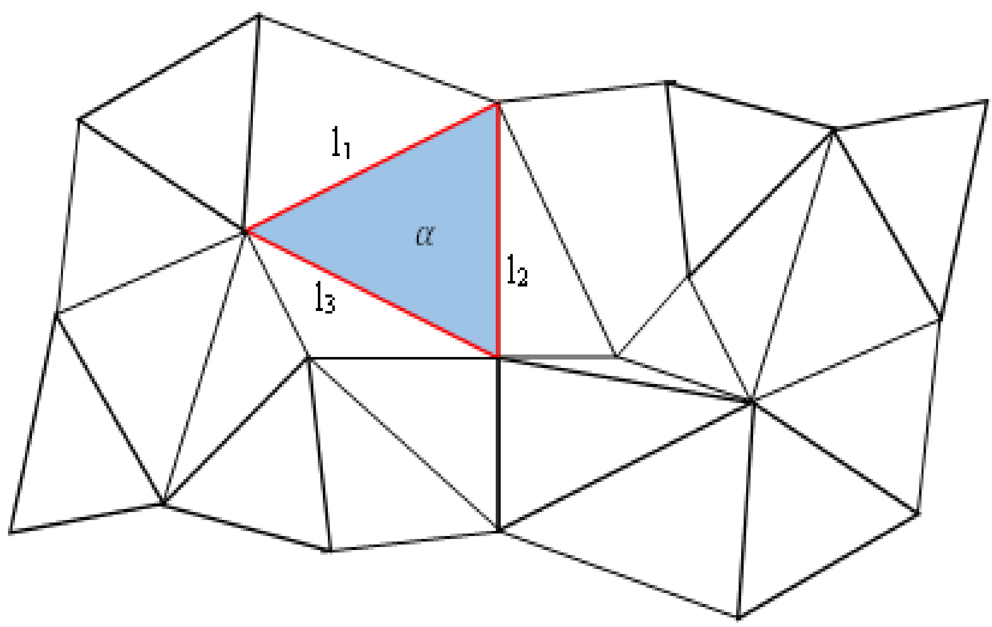
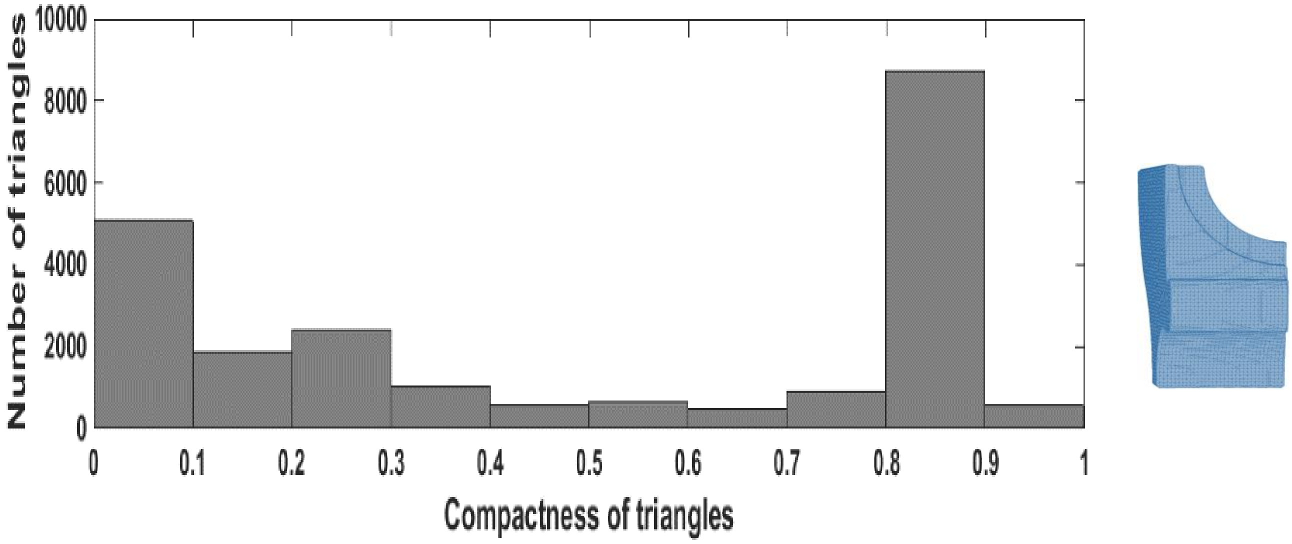


Figure 3: measure of the triangle compactness

In figures 4,5,6,7, we present the compactness of the triangles of the two meshes. In each figure, the first line presents the reconstructed mesh from the original point cloud. The second line presents the simplified point cloud. Note that, the evaluation of the mesh quality is achieved by the compactness of the triangles.

(a)



(b)

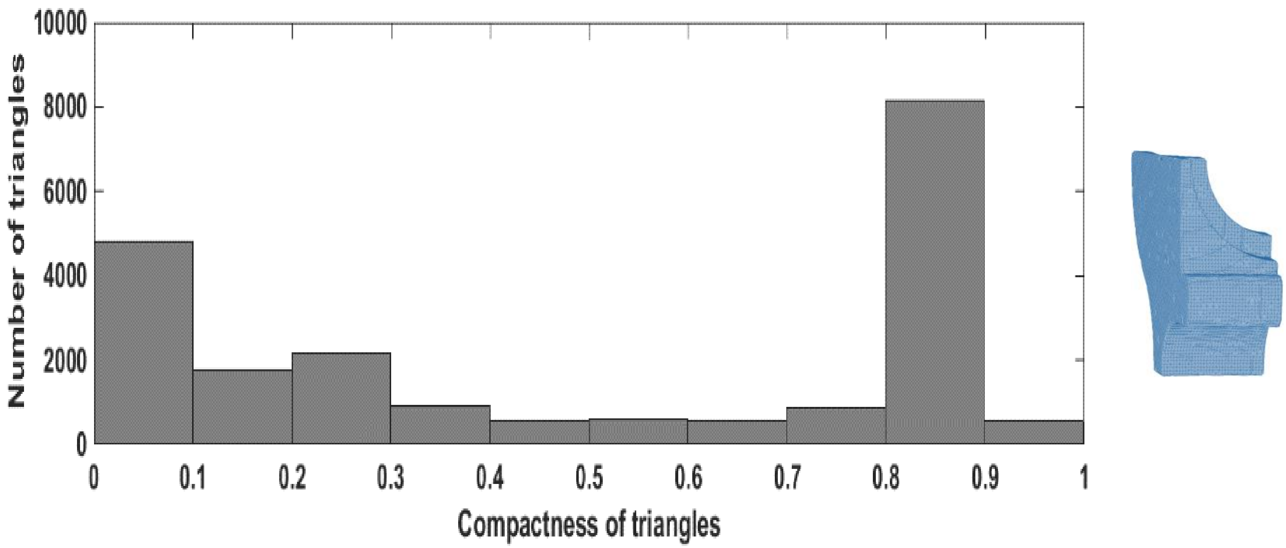
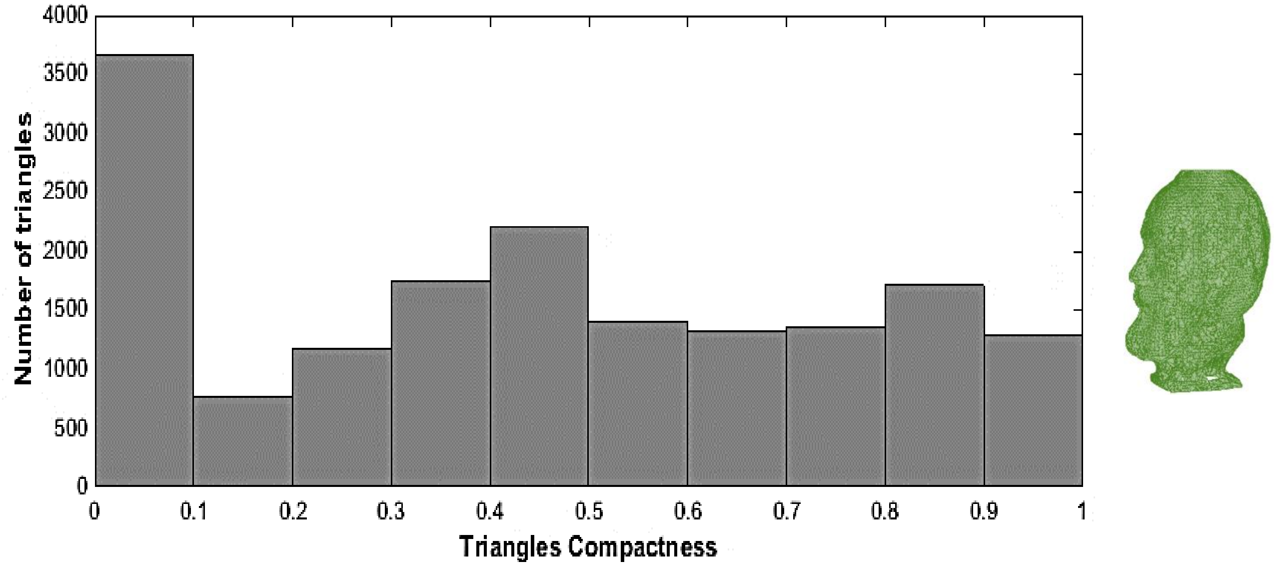


Figure 4: Comparison of Fandisck mesh quality: a) Original point cloud, b) simplified point cloud

(a)



(b)

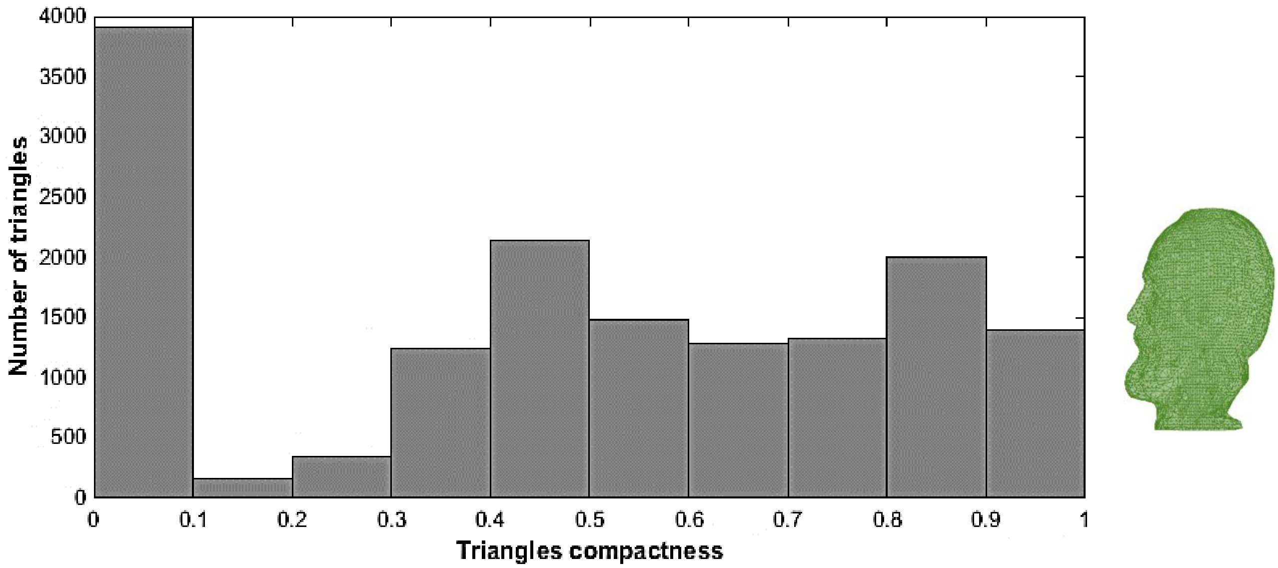
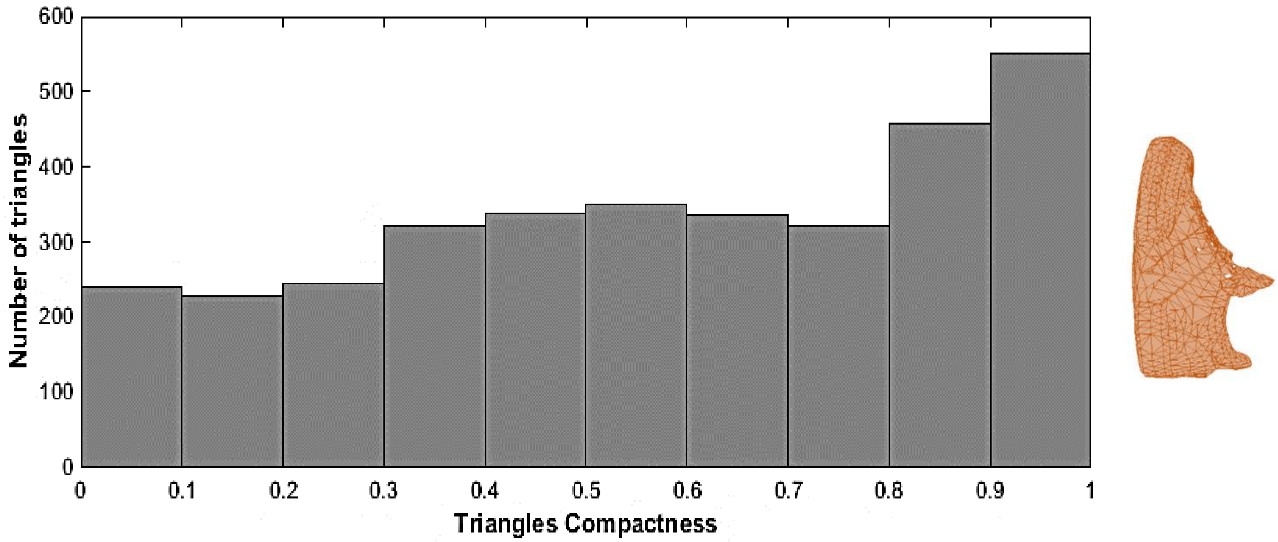


Figure 5: Comparison of Hugo mesh quality: a) Original point cloud, b) simplified point cloud

(a)



(b)

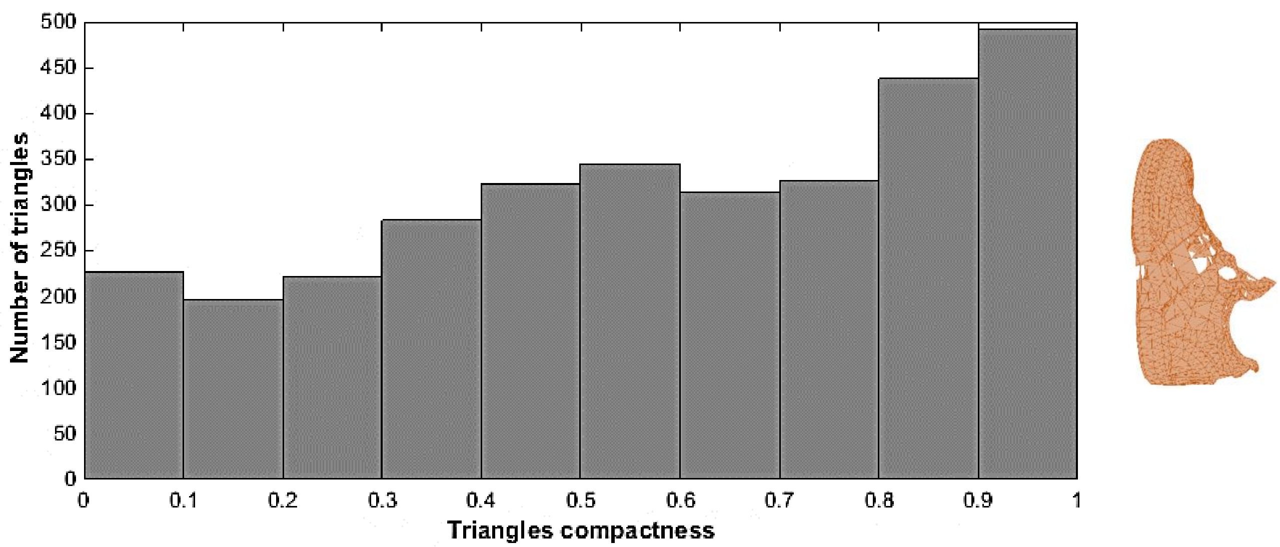
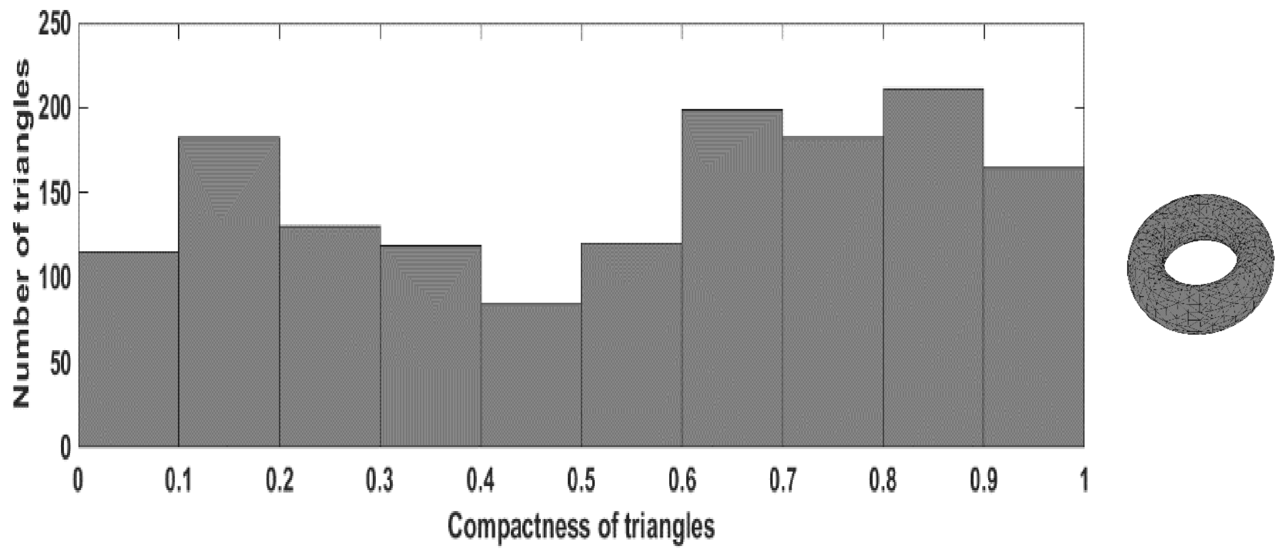


Figure 6: Comparison of Tennis shoe mesh quality: a) Original point cloud, b) simplified point cloud

(a)



(b)

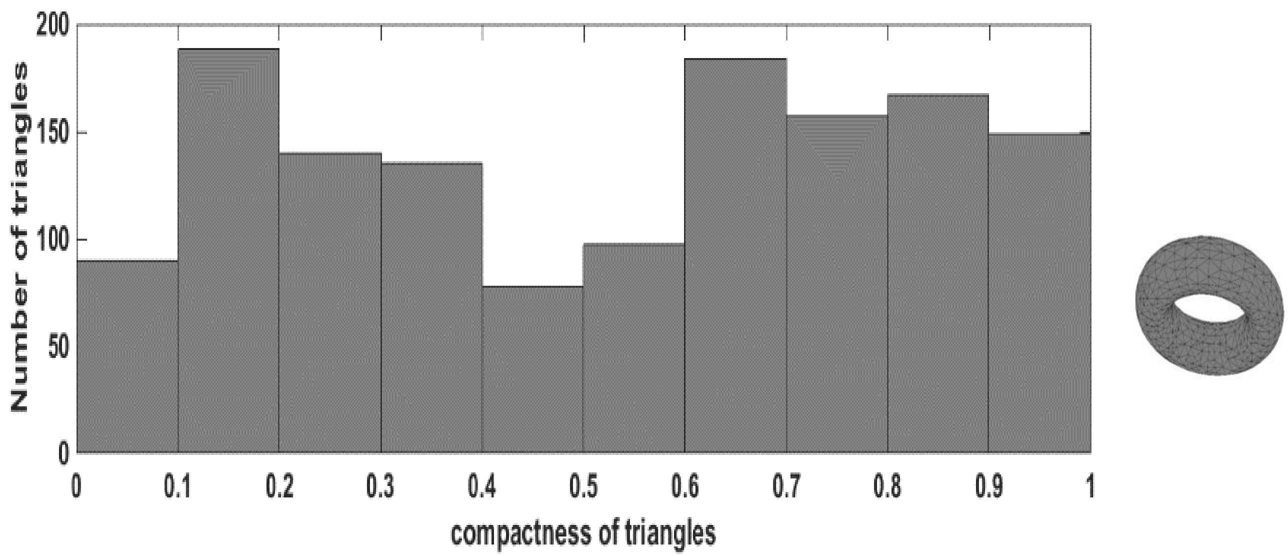


Figure 7: Comparison of torus mesh quality: a) Original point cloud, b) simplified point cloud

Table 1 shows the percentage of triangles with a compactness greater than or equal to 0.6 for each 3D model. We find that the percentage of the compactness is greater than 50% for most simplified point clouds of the 3D models. Thus, we observe that the value of compactness of the simplified point cloud is superior than the original point cloud, except for the first case (Torus object) where the compactness is less than the original. For all calculations, we take the value of the threshold s equals to 0.001.

Table 1: Mesh Compactness of Different Sample Points.

Object	Number of points		Triangles with a compactness≥ 0.6 (%)	
	Original	Simplified	Simplified	Original
Torus	800	752	47.44	50.23
Tennis shoe	1840	1692	50.45	49.13
Hugo	8281	7661	39.31	34.06
Fandisk	11984	11257	48.31	47.78

5.3. Comparison with other simplification method

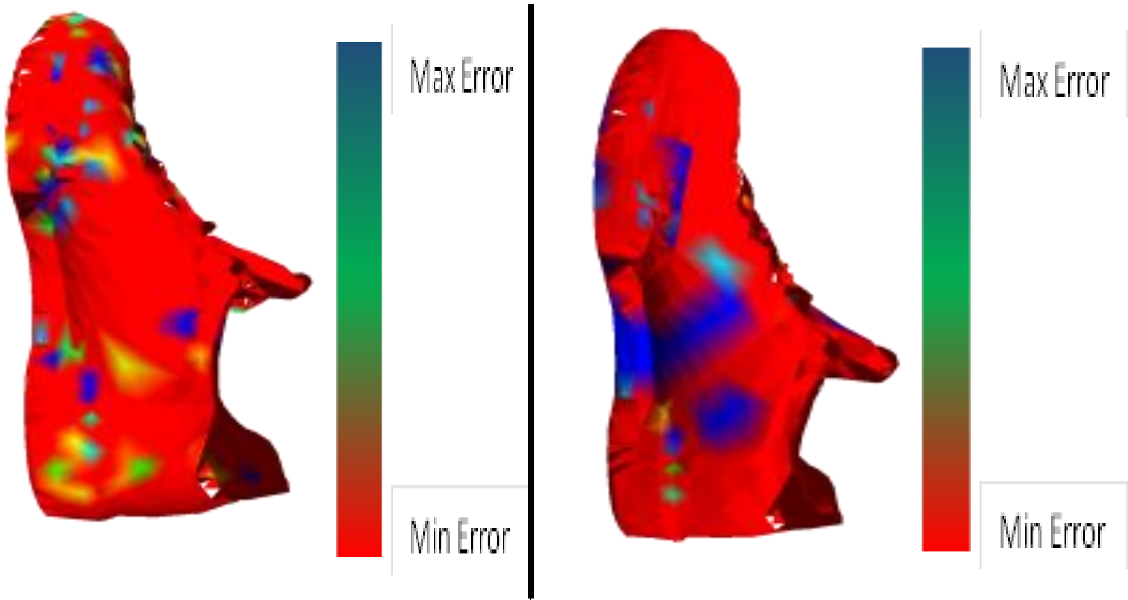
In this section, we compare our simplification method with that of R. Allègre et al. [11]. The latter method is based on two main criteria: a topological criterion ($\rho_{top} = 0.9$) and a geometric criterion ($\rho_{geo} = 1$). To simplify the R. Allègre method, it needs the estimation of normal vectors which is not the case in our method. On the other hand, the comparison of the robustness of the two methods is based on a metric of the approximation errors used by Cignoni [25].

Table 2 shows the numerical results obtained by the implementation of the two simplification methods. These results are related to Tennis-shoe model. The main results are average error, maximal error and root mean square error (RMS). Fig. 6 presents the difference between original and simplified meshes, using Hausdorff distance for the two methods. Note that, it is a red-green-blue map, so red is minimal and blue is maximal. In our case, red means zero error and blue high error.

Table 2: Comparison of our Simplification Method And Allègre's Method: Tennis-Shoe Mesh

Methods	**Max Error**	**RMS**	**Average Error**
Our simplification method	0.010968	0.001453	0.000471
Allègre's method	0.049468	0.004108	0.000726

Errors are measured as percentages of the datasets bounding box diagonal (12.53).



a) Our simplification method

b) Simplification Method of Allège et al.

Figure 8: difference between original and simplified tennis-shoe mesh

In association with the results above, the method of Allègre et al. has recorded in general the worst result in terms of error as table 2 and fig. 6.b indicate. As expected, our simplification method gives the best results in terms of average error, max error and RMS error, as table 2 indicate. In this article, we have presented a method of point cloud simplification. This technique is based on Shannon's entropy. The calculation of the compactness shows that the surfaces obtained after simplification are close to the original surfaces. Furthermore, the measurement of the error between the simplified surface and the original surface proves that our method in terms of accuracy is accurate compared to the Allègre’s method.

We have implemented our simplification method under MATLAB. The calculations are performed on a machine with an i3 CPU, 3.4 GHz, and 2GB of RAM.

6. CONCLUSION

Throughout this work, we have proposed a new approach for simplification of 3D point cloud using the Shannon’s entropy with a K nearest neighbors estimator which allows only the consideration of relevant data. In the first stage, we have applied our simplification algorithm to different point clouds with different densities. Subsequently, to validate the obtained results, on the one hand, we have compared our method with that of Allègre. On the other hand, we have measured compactness of simplified meshes. From these results, we conclude that our approach is efficient and robust.

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